FAST NU / NUST NET Entry Test:
Advance Math Most Important MCQs

Bonus Q. If $\vec{v} = a\hat{i} + a\hat{j} + a\hat{k}$ is a unit vector, then a = ______.

a =
- a) 1/3
- b) 0
- c) 1
- d) -1
- e) $1/\sqrt{3}$

$\cancel{\sqrt{1+1+1}}$

$\sqrt{a^2+a^2+a^2}$

$\sqrt{3a^2}=1$

$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = \sqrt{x^2+y^2+z^2}$

$\sqrt{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$

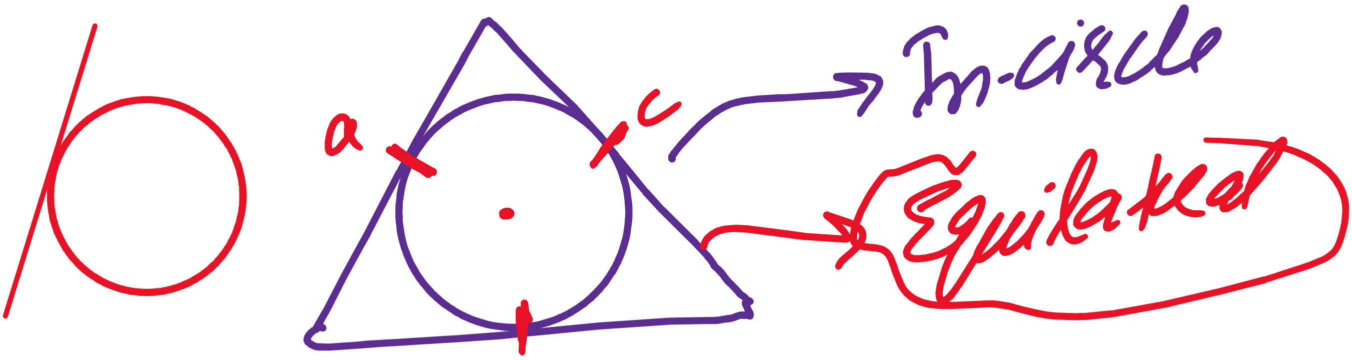
$\sqrt{3 \times \frac{1}{\sqrt{3}}}$

$\sqrt{3a^2} = 1$

$a\sqrt{3} = 1$

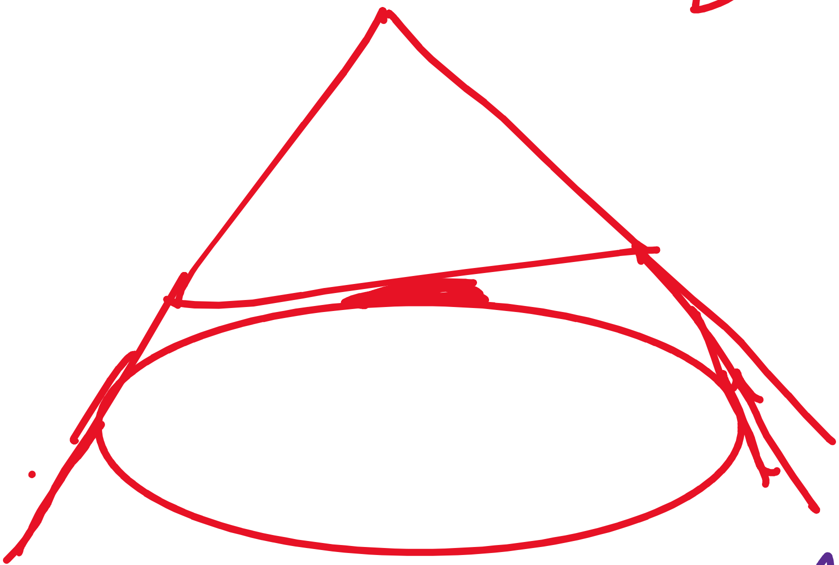
$a = \frac{1}{\sqrt{3}}$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$



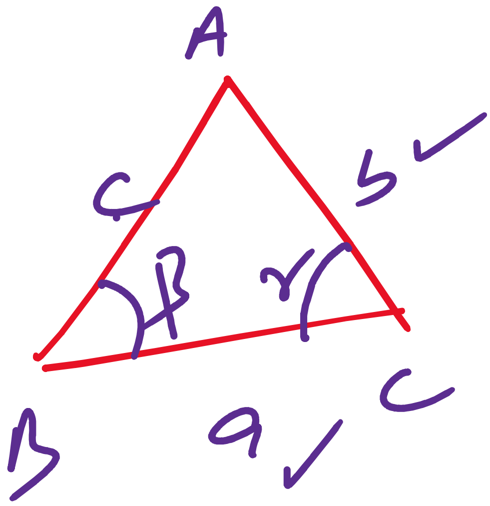
D

Ex-circle / Escribed ~~Ex-circle~~



$\Delta = \frac{1}{2}ab\sin\gamma$

$= \frac{1}{2}ac\sin\beta$ ✓



Equilateral Δ

$a = b = c$ ✓

$r = \frac{\Delta}{s} \Rightarrow$

✓

$s = \frac{3a}{2}$

$r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$

$S = \frac{3}{2}$ ✓ $r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$

$$\boxed{\Delta = \frac{\sqrt{3}a^2}{4}}$$

$$\boxed{r_1 = r_2 = r_3 = \frac{\sqrt{3}a}{2}}$$

$$R = \frac{abc}{4\Delta} = \frac{a}{\sqrt{3}}$$

✓ 3 vow   AUE

Q69. How many words can be formed from the letters of the word "DAUGHTER" if vowels are to be placed at second, fourth and sixth places?

a) 740

b) None of them

c) 720

d) 865

5 ← $\underline{\ }$ $\underline{V_1}$ $\underline{4}$ $\underline{V_2}$ $\underline{3}$ $\underline{V_3}$ ...

$$\boxed{3 \times 2 \times 1}$$

$3! = 6$ ways

$\times$

$5! = 120$ $5\times4\times3\times2\times1$

720

$A \cap B$ $\cancel{7}, 8, \cancel{9}, 10, \cancel{11}, 12$

$A \cap B$

$\{7, 8, \cancel{9}, 10, \cancel{11}, 12\}$

Q71. Two dice are thrown simultaneously. Event A: the sum of dots is greater than 6. Event B: the sum of dots is even. P(A ∩ B) equals:

a) 5/6
b) 1/2
c) 9/36
d) 21/36

$\Delta = \frac{1}{2}ab$

$\{1, 2, 3, 4, 5, 6\}$ $\{1, 2, 3, 4, 5, 6\}$

Sum

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2+6
6+2
3+5
5+3
4+4

$6^3 = 216$

8 → 5 times
10 → 3 times
12 → 1 time

$$P(A \cap B) = \frac{9}{36} = \frac{1}{4} = 0.25$$

$6^2 = 36$ $\qquad$ 6^n

Even $\longrightarrow$

Coin = {H, T} $\quad 2^1 = 2$

$2\ H = 2^2 = 4$

$2^3 = 8$

	Closure	Assoc.	Ident.	Inverse
Groupoid	✓	✗	✗	✗
Semi-Group	✓	✓	✗	✗
Monoid	✓	✓	✓	✗
Group	✓	✓	✓	✓

Q67. Let H be a non-empty set and * a binary operation in H. An element e of H is an identity element with respect to * if for all a ∈ H, _____.

a) a * e = 1

b) a * e = a

identity element with respect to * if for all a ∈ H, _____.

a) a * e = 1
b) a * e = e
c) a * e = a³
d) a * e = a
e) a * e = a²

a∈H

a + 5

a × 5

+ → 0

5+0=5

2+0=2

× → '1'

5×1=5

6×1=6

5+0=5

5×1=5

$a * e = e * a = a$

$a * b = e$

$5 \times \frac{1}{5} = 1$

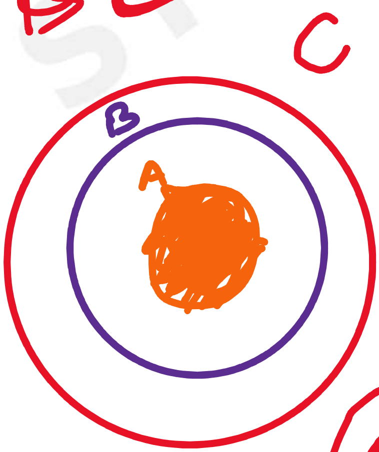
$-5 + 5 = 0$

Q65. If A ⊂ B and B ⊂ C, then A ∩ C = ______.

a) A′
b) C
c) ϕ (Null set)
d) B
e) A

$1 < 2 < 3$

$A \subset B, \; B \subset C$



$A \subset A$

$A \cap C$

One

Q63. Evaluate the determinant:

$$\begin{vmatrix} y^2z & z^2x & x^2y \\ x/y & y/z & z/x \\ y/x & z/y & x/z \end{vmatrix}$$

a) 1
b) 0
c) xyz
d) $x^2y^2z^2$
e) -1

$\frac{xyz}{xyz}$

R

R_3

y^2z

$\frac{x}{\cancel{y}} \times xyz$

$\frac{z}{\cancel{y}} \times xyz$

Q63. Evaluate the determinant:

| y²z z²x x²y |
| x/y y/z z/x |
| y/x z/y x/z |

a) 1

b) 0

c) xyz

d) x²y²z²

e) -1

$\frac{x}{y}, \frac{z}{x}, \frac{y}{x}$

$x = 16, y = 8$

$x = 4$
$y = 2$ $z = 1$

$x = 4$

$\frac{2}{4} = \frac{1}{2}$

$x = 1, y = 2, z = 3$

$\frac{1}{2}, \frac{1}{3}$

$\frac{2 \times 3}{2 \times 3}$

Q64. The value of the determinant is:

| x²-y² y²-z² z²-x² |
| y²-z² z²-x² x²-y² |
| z²-x² x²-y² y²-z² |

a) -1

b) 0 ✓

c) 1

d) (x-y)(y-z)(z-x)

e) (x²-y²)(y²-z²)(z²-x²)

$\cancel{x^2-y^2, y^2-z^2, z^2-y^2}$

0

Q58. [a b b] = _____.

a) 0

[a b k] scalar

Q58. [a b b] = _____.

a) 0

b) -1

c) k

d) i

e) 1

$\begin{vmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = [a\ b\ k] \rightarrow$ scalar Tm

Q59. Two or more vectors are said to be coplanar if they are parallel to the same _____.

a) Vector

b) Line

c) Magnitude

d) Plane

e) Scalar

$a \cdot (\underline{b} \times \underline{c})$

$a \cdot b \times c = a \times b \cdot c$

$b \cdot c \times a$

$b \times c \cdot a$

$c \cdot a \times b$

$c \times a \cdot b$

$\underline{c} = c_1\hat{i} + c_2\underline{j} + c_3k$

$a \rightarrow$

b

c

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\underline{a} = \underline{i} + \underline{j} + \underline{k}$, $\underline{b} = -2i + 3\underline{j} + 4\underline{k}$

$\underline{c} = \underline{i} - \underline{j} + \underline{k}$

$$[\underline{a}\cdot\underline{b}\times\underline{c}] = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 3 & 4 \\ 1 & -1 & 1 \end{vmatrix} =$$

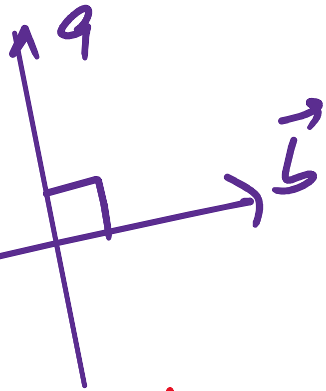
Volume of Tetrahedron $= \frac{1}{6}|\underline{a}\cdot\underline{b}\times\underline{c}|$

Volume of Parallelepiped $= |\underline{a}\cdot\underline{b}\times\underline{c}|$

Q60. For what value of a will the vector [1, 2a, -4] be perpendicular to the vector [-3, 1, 2]?

a) 11/2
b) -3
c) 4
d) 2/11
e) -1

$\vec{a}\cdot\vec{b} = \underline{a}\,\underline{b}\cos 90^\circ$



$-3 + 2a - 8 = 0$

$2a = 11$

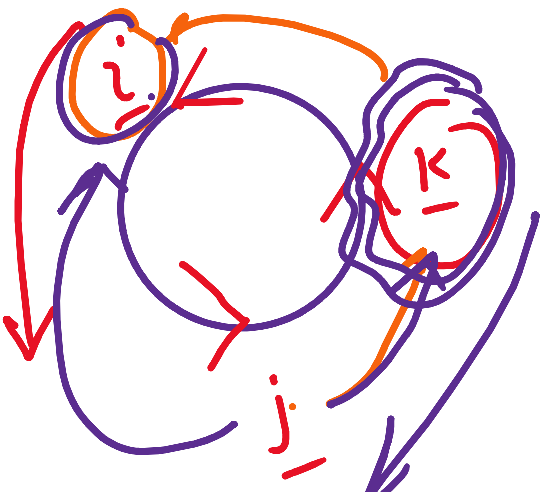
$a = 11/2$

Q61. i × j = _____ . K

a) -j × i
b) k × i
c) zero
d) j × i
e) k × i

$-\,j \times i$

$-(-k)$



d) j × i
e) k × j

$i \times j = k$

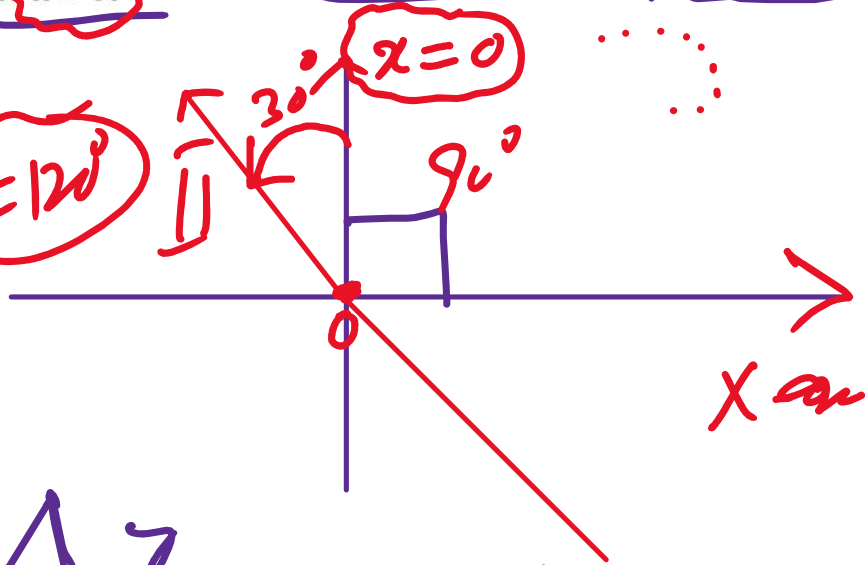
(0,0)

Q57. What is the equation of the line passing through the origin and making an angle of 30° anticlockwise with the line x = 0?

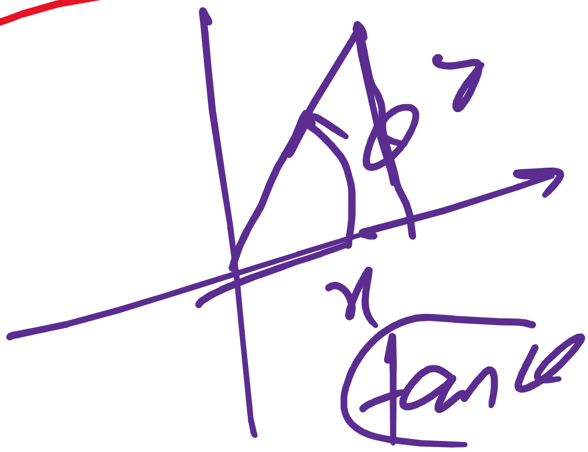
a) y = √3x
b) y = -√3x
c) 2y = -√3x
d) x = -√3y
e) 2y = √3x

$x=0$

$\theta = 120^\circ$



$y = mx$



$\tan\theta = \text{slope} = \frac{\Delta y}{\Delta x}$

S | A
T | C

$m = \tan 120^\circ$

odd

II

$\tan(90^\circ + 30^\circ)$

Allied

$= -\cot 30^\circ = -\frac{1}{\tan 30^\circ}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$= -\frac{1}{\frac{1}{\sqrt{3}}}$

$= -1 \div \frac{1}{\sqrt{3}} = -1 \times \frac{\sqrt{3}}{1}$

$T_5 = T_{4+1}$ → r

Q41. Which term in the expansion of $(x/2 + 2/x^2)^{12}$ is independent of x?

a) 6
b) 5
c) 13
d) 1

$\left(\frac{x}{2} + \frac{2}{x^2}\right)^{12}$ — x^0

$r = \frac{np}{p+q}$

$= \frac{12 \cdot 1}{3} = 4 = r$

$T_{r+1} = \binom{n}{r} a^{n-r} \cdot b^r = (a+b)^n$

$T_5 = T_{4+1}$

$\left(ax^p + \frac{1}{bx^q}\right)^n$

$$r = \frac{mp}{p+q}$$

Q56. If x²/81 + y²/121 = 1 for an ellipse, then the length of latus rectum is:

a) 172/11
b) 162/11
c) 163/21
d) 165/21
e) 242/81

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$a > b$

$$\frac{2b^2}{a} = \frac{2 \times 81}{11}$$

$a^2 = 121 \Rightarrow a = 11$

A Level NET

Q55. The eccentricity of the hyperbola x²/25 - y²/36 = 1 is:

a) 5/√61
b) √61/5
c) √61/9
d) -√61/5
e) -9/√61

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a^2 = 25$
$b^2 = 36$

$a = 5$

$$\frac{x^2}{25} - \frac{y^2}{36} = 1$$

x^2 $2x$ 30

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$c = \sqrt{a^2+b^2}$

$$e = \frac{\sqrt{a^2+b^2}}{a} = \frac{c}{a}$$

$=$

Q54. If f(x) = √(x - 1)/x, what is the domain of f(x)?

a) All real numbers less than or equal to -1
b) All real numbers except 0
c) All real numbers greater than or equal to 1
d) All real numbers less than or equal to 1
e) All real numbers greater than or equal to -1 but less than or equal to 1

x 0

$0 \cdot x$

$$f(x) = \frac{\sqrt{x-1}}{x}$$

$x \cdot$ $\frac{1}{0}$

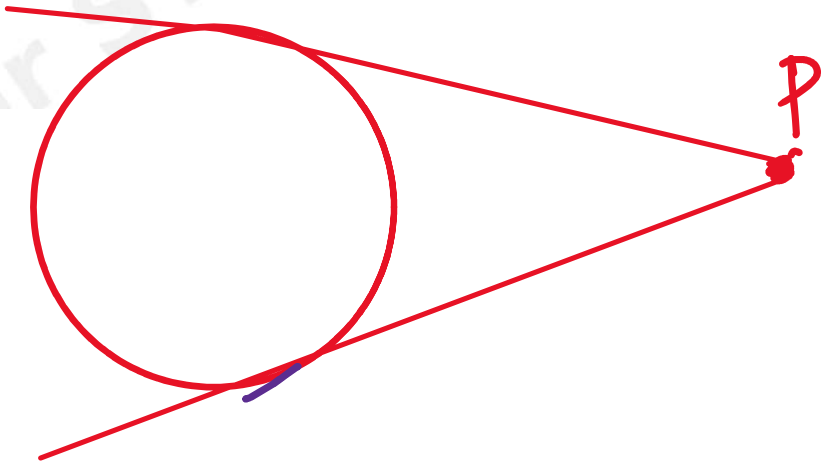
$\sqrt{0} = 0$

$x - 1 \geq 0$

$x \geq 1$

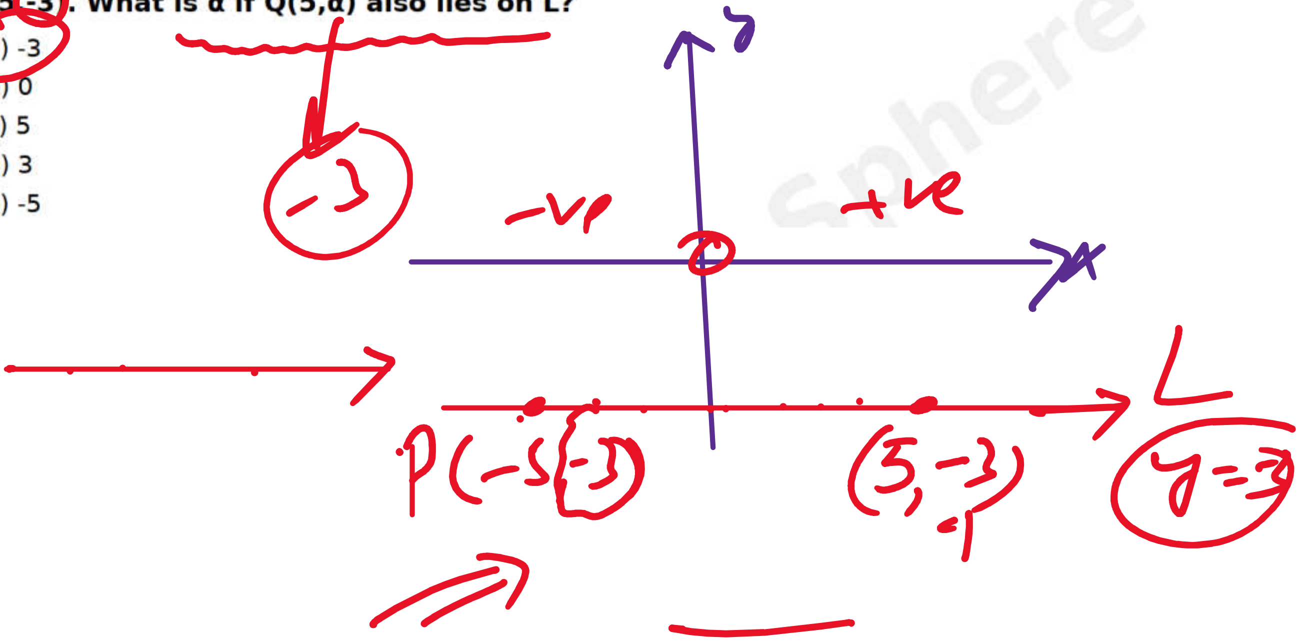
Q53. Two tangents drawn from a point P to the circle $x^2 + y^2 = 121$ may meet at a point other than P which lies at ______.

a) No such point exists
b) $x = 11$
c) $x = \infty$
d) $y = 11$
e) $y = \infty$



Q52. A line L is perpendicular to the y-axis and the coordinates of a point P on L are (-5,-3). What is α if Q(5,α) also lies on L?

a) -3
b) 0
c) 5
d) 3
e) -5



Q51. Two lines $ax^2 + 2hxy + by^2 = 0$ will be imaginary if ______.

a) $h^2 < ab$
b) $h^2 = ab$
c) $ab = 0$
d) $h^2 > ab$
e) $h^2 = -ab$

$$\sqrt{h^2 - ab}$$

$$h^2 - ab < 0$$

$$h^2 < ab$$

Q50. The point P(0,b) lies inside the circle $x^2 + y^2 - 2 = 0$ if b = ______.

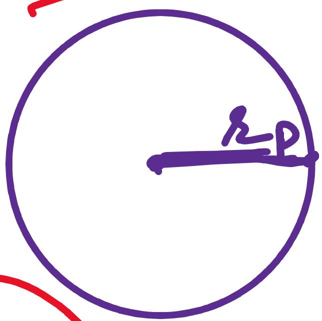
Q50. The point P(0,b) lies inside the circle $x^2 + y^2 - 2 = 0$ if b = ______.

a) $-\sqrt{2}$
b) 2
c) -2
d) $\sqrt{2}$
e) -1

$x=0$ $y=b$

$0+1<2$

$(-\sqrt{2})^2 = 2$: a



$x^2+y^2=2$

$x^2+y^2=(\sqrt{2})^2$

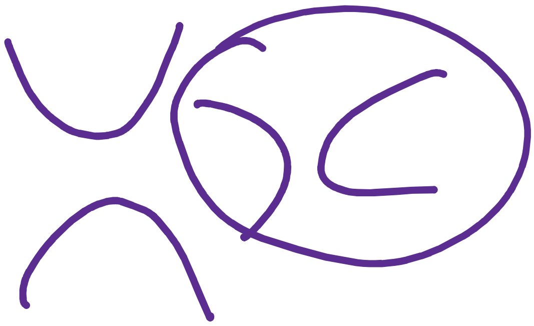
$r=\sqrt{2}$

$x_1^2+y_1^2<a^2$

Q49. A conic is the set of points such that the distance of each from a fixed point bears a constant ratio greater than one to its distance from a fixed straight line. This conic is ______.

a) Ellipse
b) Directrix
c) Parabola
d) Circle
e) Hyperbola

$e=\frac{|PF|}{|PL|}>1$



Parabola: $\frac{PF}{PL}=1$

$0<e<1 \to$ Ellipse

$e=0 \to$ circle

$i^2=-1$

Q46. The imaginary term of $-i^2(2 - i)$ is ______.

a) 2i

$2(-i^2) - i(-i^2)$

Q46. The imaginary term of $-i^2(2 - i)$ is ______.

a) 2i
b) 2
c) $2i^2$
d) -i

$\rightarrow 2(-i^2) - i(-i^2)$

$(-(-1))$

$+1$

Q47. If one root of $3x^2 - 6hx + h^2 - 25 = 0$ is zero, then the value of h is ______.

a) 4
b) 5
c) -4
d) -5
e) ±5

$x = 0$

-2

$h^2 = 25$

$h = \pm 5$

Trigonometry and Trigonometric Functions

Q1. What is the period of $f(x) = (1/2)\cos 2x$?

a) 6π
b) π
c) π/2
d) 4π
e) 2π

$A\cos(Bx + C) + D$

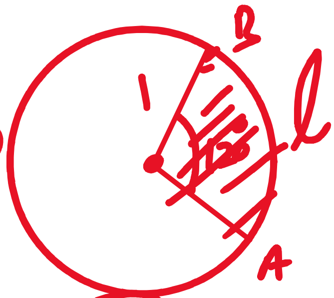
$P = \frac{2\pi}{|B|} = \frac{2\pi}{2} = \pi$

$2x = 2\pi \quad x = \pi$

Q2. What is the length of the arc of a circle of radius 1 cm and angle 120°?

a) 1.86 cm
b) 2.09 cm
c) 22.9 cm
d) 1.5 cm

$l = r\theta \rightarrow$ radians



$\overset{\frown}{AB}$

$\frac{1}{2}r^2\theta$ ↓ Area

$\theta = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

$l = \frac{2}{3} \times \frac{22}{7} = \frac{44}{21}$

Q3. Find the length of the arc whose radius is 9 cm and central angle is 360°.

a) 56.55 cm
b) 69.16 cm

2π

Q3. Find the length of the arc whose radius is 9 cm and central angle is 360°?

a) 56.55 cm
b) 69.16 cm
c) 3140 cm
d) 65.54 cm
e) 74.13 cm

2π

$$l = 2\pi \times 9 = 18 \times 3.14$$

Q4. sin 30° + sin 16° = ?

a) 2 sin 14° cos 14°
b) -2 sin 28° cos 14°
c) 2 sin 7° cos 23°
d) 2 cos 17° sin 14°
e) 2 sin 23° cos 7°

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\frac{46}{2} = 23^\circ, \quad \frac{30-16}{2} = \frac{14}{2} = 7^\circ$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = \_\_\_\_$$

$$\cos A - \cos B = ?$$

$$2\sin\alpha\cos\beta = \_\_\_\_$$

$$2\cos\alpha\sin\beta = \_\_\_\_$$

$$2\cos\alpha\cos\beta = \_\_\_\_$$

$$-2\sin\alpha\sin\beta = \_\_\_\_$$

$$\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$$

$\sin(-\theta) = -\sin\theta$ odd

$\frac{1}{2}$ — $\frac{1}{2}$

Q5. If cos(30° - α) - sin(α - 30°) = 0, where 0° ≤ α ≤ 90°, then α = _____.

a) 15°
b) 45°
c) 75°
d) 30°
e) 60°

$\cos(15^\circ) + \sin 15^\circ$ ✗

$\cos(-15) = \cos 15$ → Even

75
30
—
45

b) $\cos 15 - \sin 15^\circ$

$\alpha + \beta = 90^\circ$

$\cos 30^\circ$
$\sin 60^\circ$

$\sin 30^\circ = \cos 60^\circ$

$\frac{1}{2} = \frac{1}{2}$

$\sin 40^\circ = \cos 50^\circ$

$\cos 0^\circ = 1, \quad \sin 0 = 0$

III & IV

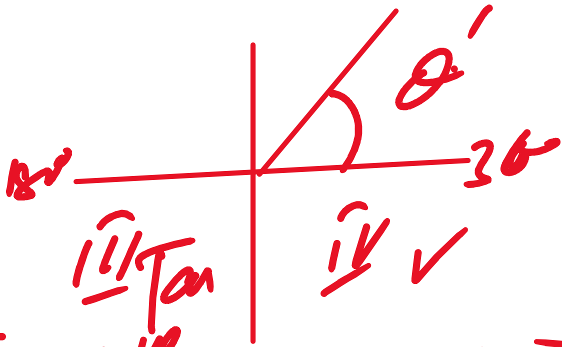
Q6. If 180° ≤ θ ≤ 360° and θ = tan⁻¹(-√3), then θ = _____.

↳ Ref.

a) 210°
b) 330°

Q6. If 180° ≤ θ ≤ 360° and θ = tan⁻¹(−√3), then θ = ______. ↳ Ref.

a) 210°
b) 330°
c) 300° ✓
d) 240°
e) 60°



$\tan 60° = \sqrt{3}$

$\tan\theta = -\sqrt{3}$

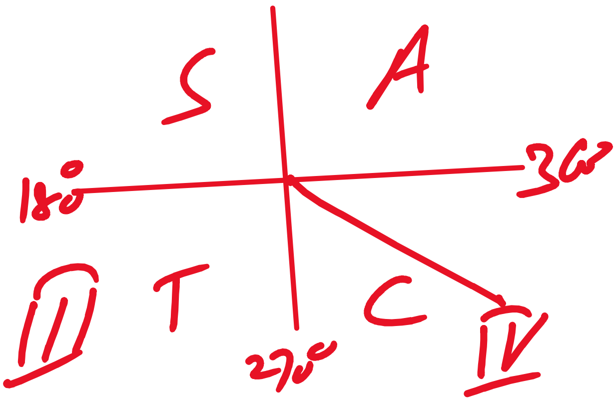
$\theta = 60°$ → Ref.

$\theta = 2\pi - 60° = 360° - 60° = 300°$

$\sin(360° - 60°) = -\sin 60° = -\sqrt{3}$

Q7. sin 300° equals ______. −ve

a) sin 120°
b) cos 120°
c) sin 240° ✓
d) cos 240°
e) cos 30°



$\sin 240° = \sin(180° + 60°) = -\sin 60° = -\sqrt{3}$

Q8. Which of the following is equal to tan θ + cot θ? $\theta = 45°$

a) (sec θ)(cosec θ)
b) 2/sec² θ
c) 1/cot θ + tan θ
d) (sin θ + cos θ)/(sin θ + cos θ)
e) tan² θ + 1

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

d) (sin θ + cos θ)/(sin θ + cos θ)

e) $\tan^2 \theta + 1$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$\Rightarrow \sec\theta \csc\theta$$

$(1-\cos^2 x)$

Q9. If 0 ≤ x ≤ 360° and 4 sin²x + 4 cos x - 1 = 0, which set contains all values of x?

a) {60°, -60°}
b) {30°, 60°}
c) {60°, 120°}
d) {120°, 240°}
e) {120°, -120°}

$\sin 60° = \frac{\sqrt{3}}{2} \Rightarrow \frac{3}{4}$

$60° \Rightarrow \cancel{4} \times \frac{3}{\cancel{4}} + \cancel{4}^{2} \times \frac{1}{\cancel{2}} - 1$

$240° \Rightarrow \sin(180° + 60°)$

$-\sin 60° = \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

$\cos(180° + 60°) = -\cos 60° = -\frac{1}{2}$

$\cancel{4} \times \frac{3}{\cancel{4}} + \cancel{4}^{2}\left(-\frac{1}{\cancel{2}}\right) - 1$

$3 - 2 - 1 = 3 - 3 = 0$

Q10. If cos x - 1 = -2 sin((x+y)/2) sin((x-y)/2), where x ≥ 0 and y ≤ 90°, then y = _____.

a) 45°
b) 30°
c) 90°
d) 60°
e) 0°

$-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

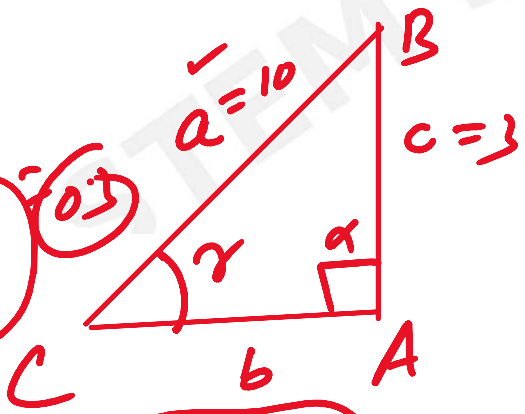
$\cos x - \cos y =$

$\cos y = 1$

$\to 0°$

Q11. In any triangle ABC, a = 10, c = 3, α = 90°, find sin γ.

a) 0.2
b) 0.9784
c) 1.2745
d) 0.8485
e) None of these



$\sin\gamma = \frac{c}{a} = \frac{3}{10} = 0.3$

$\sin\theta = \frac{P}{H}$, $\cos\theta = \frac{B}{H}$

$\tan\theta = \frac{P}{B}$

Q12. The slope of the tangent to the curve y = e^(ln(sin x)) at x = 60° is _____.

a) 0.5
b) 0.866
c) 10
d) e^(ln 0.5)
e) e^(ln 0.866)

$e^{\ln(\sin x)}$

$\ln e = 1$

$\cot x = \frac{5}{10}$ | $y = \sin x$

$e^{\ln(1)} = 1$

$\log_{10} x = 5$

$x = 10^5$

$y = \sin x$

$y' = \cos x$

$x = 60°$

$= \cos 60 = 1/2$

Q13. If y = f(x) = -(5/2) sin(-√3 x + 7), then the range of f(x) is ______.

a) 5/2 ≤ y ≤ -5/2

b) -1 ≤ y ≤ 1

c) -∞ < x < ∞

d) -√3 ≤ y ≤ √3

e) -5/2 ≤ y ≤ 5/2

$-1 \le \sin(\ ) \le +1$

$(-5/2)$

$[-\frac{5}{2}, \frac{5}{2}]$

$-\frac{5}{2}$

$-\frac{5}{2} \le y \le \frac{5}{2}$

Q14. If y = f(x) = -√3 tan 2x, then the range of f(x) is ______.

a) -2 ≤ y ≤ 2

b) -60° ≤ y ≤ 60°

c) -∞ < y < ∞

d) -√3 ≤ y ≤ √3

e) R - {x | x = (2n+1)π/4, n ∈ Z}

$\tan(-90°) = -\infty$

$\tan(90°) = +\infty$

Q15. If f(x) = e^(cos x) on the interval 0 < x < π/6, then f(x) is ______.

a) Neither increasing nor decreasing

b) Information is incomplete

c) Increasing as well as decreasing

d) Decreasing

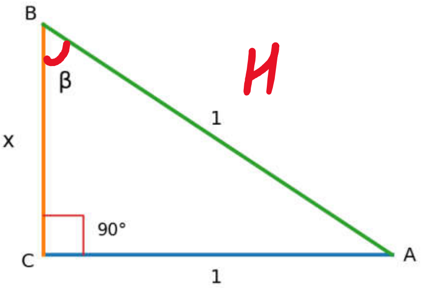
e) Increasing

$0 < \theta < 30°$

$f'(x) = e^{\cos x} \times (-(\sin x))$

$f'(x) = -\{e^{\cos x} \cdot \sin x\} < 0$

Q16. In the given figure, if sec β = ∞, then x = ______.



B H P

$\cos\beta = \frac{B}{H}$

$\sec\beta = \frac{H}{B} = \frac{1}{x} = \infty$

- a) π/2
- b) 0
- c) √2
- d) π/4
- e) 1

Differential Calculus

Q17. If y = 5^x t, then dy/dx = ?

- a) 4
- b) -14
- c) 1
- d) 5^x ln 5
- e) t(5^x ln 5)

$y = 5^x \, \underline{t}$ const.

$y' = t(5^x \ln 5)$

Q18. Let y = cos x + cos 2x, then dy/dx = ?

- a) 2 cos x(-sin x)
- b) (1/2) cos 2x
- c) -sin x - 2 sin 2x
- d) sin x(2 sin 2x)

$-\sin x - \sin 2x \times \frac{d}{dx}(2x)$

$\boxed{-\sin x - 2\sin 2x}$

Q19. If y = 1/ln(1/x), then d^2y/dx^2 = _____.

a) $-\{ 1/[x^2(\ln x)^2] + 2/[x^2(\ln x)^3] \}$
b) $-1/x^2$
c) $-1/x^3$
d) $1/[(\ln x)^2 x]$
e) $-\{ 1/(x \ln x) + 2/[x^2(\ln x)^2] \}$

$$y = \frac{1}{\ln\left(\frac{1}{x}\right)} = \frac{1}{(\ln x^{-1})}$$

$$y = \frac{1}{-\ln x} = -(\ln x)^{-1}$$

$$y' = -(-1)(\ln x)^{-2} \times \frac{1}{x}$$

$$\boxed{y' = (\ln x)^{-2} * \frac{1}{x}}$$

$$x^{-1}$$

$$y'' = (\ln x)^{-2}\left(\frac{-1}{x^2}\right) + (-2)\frac{(\ln x)^{-3}}{x} \cdot \frac{1}{x}$$

$$= -\frac{1}{x^2(\ln x)^2} - \frac{2}{x^2(\ln x)^3}$$

20)

$$\frac{d}{dx}\left[(g(x))^m\right]^m = \_\_\_\_\_\_$$

$$m\left[(g(x))^m\right]^{m-1} \times$$

Options

- a) $m[g(x)]^{m-1}$
- b) $m\left[(g(x))^m\right]^{m-1}\left[m(g(x))^{m-1}\right]\frac{d}{dx}(g(x))$
- c) $m[g(m-1)x]g'(x)$
- d) $m\left[[g(x)]^{m-1}\right]^{m-1}$
- e) $n\left[g(m-1)^{n-1}\right]g'(x)$

$$\frac{d}{dx}\left((g(x))^m\right)$$

$$m\,(g(x))^{m-1} \times \frac{d}{dx}(g(x))$$

$m\,(g(x)) \times \frac{d}{dx}(\ldots)$

21) Q) If $y=\left(\sin\frac{1}{x}\right)^{-1}$ then $y'=?$

- a) $\frac{\cos(1/x)}{(x\sin(1/x))^2}$ ✓
- b) $\cos\frac{1}{x}$
- c) $(-1)(\sin(1/x))^{-1}(\cos(1/x))$
- d) $\frac{\cos(1/x)}{(\sin(1/x))^2}$
- e) $\cot\frac{1}{x}$

$(-1)\left(\sin\frac{1}{x}\right)^{-2} \times \cos\left(\frac{1}{x}\right)\frac{-1}{x^2}$

$= \frac{\cos(1/x)}{\left(x\sin\frac{1}{x}\right)^2}$

$x^{-1} = (-1)x^{-2} = \frac{-1}{x^2}$

$\frac{d}{dx}ix = i$

Q22. If $y=xe^{ix}$, where $i=\sqrt{-1}$, then $y''=$_____.

- a) $2e^{ix}$
- b) $(2i-x)e^{ix}$
- c) $-e^{ix}$
- d) $(i+ix-x)-e^{ix}$
- e) $-xe^{ix}$

$y' = xe^{ix}(i)+e^{ix}$

$y' = e^{ix}(ix+1)$

$y'' = e^{ix}(i)+e^{ix}\times i(ix+1)$

$= e^{ix}(i+i^2x+i)$

$= e^{ix}(2i-x)$

Q23. d/dx [f(x²)g(x)] = ?

a) $f(x^2)g(x) + f(x^2)g(x)$

b) $f'(x^2)g'(x)$

c) $2xf'(x^2)g(x) + f(x^2)g'(x)$

d) $2xf'(x)g(x) + f(x)g'(x)$

e) $f(x^2)g(x)$

$f(x^2)g'(x) +$

$f'(x^2) \times 2x \cdot g(x)$

Q24. If $y = \cos^{-1}(x^{-1})$, then $y' = ?$

- a) $1/(-\sin x)$
- b) $\sec x \tan x$
- c) $-1/\sqrt{1-x^2}$
- d) $1/\sqrt{1-x^2}$
- e) $1/[x\sqrt{x^2-1}]$

$y = \cos^{-1}\left(\frac{1}{x}\right)$

$y' = \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \times \frac{d}{dx}(x^{-1})$

$= \frac{\cancel{-}1}{\sqrt{\frac{x^2-1}{x^2}}} \times \frac{\cancel{-}1}{x^2}$

$= \frac{1}{\frac{\sqrt{x^2-1}}{x}} \times \frac{1}{x^2}$

$= \frac{\cancel{x}}{\sqrt{x^2-1}} \times \frac{1}{x\cancel{^2}}$

$= \frac{1}{x\sqrt{x^2-1}}$

$$= \frac{\qquad}{x\sqrt{x^2-1}}$$

Q25. $\int \sqrt{1-\sin x}\, dx =$ ______.

- a) $-2(\sin(x/2)+\cos(x/2))+C$
- b) $\sqrt{1+\cos x}+C$
- c) $\sqrt{x+\cos x}+C$
- d) None of these
- e) $\sin x - \cos x + C$

$\sin 2x = 2\sin x \cos x$

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$\sqrt{1-\sin x}$

$= \sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}$

$\int \sqrt{\left(\cos\left(\frac{x}{2}\right)-\sin\frac{x}{2}\right)^2}\, dx$

$= \frac{\sin\frac{x}{2}}{\frac{1}{2}} + \frac{\cos\frac{x}{2}}{1/2} + C$

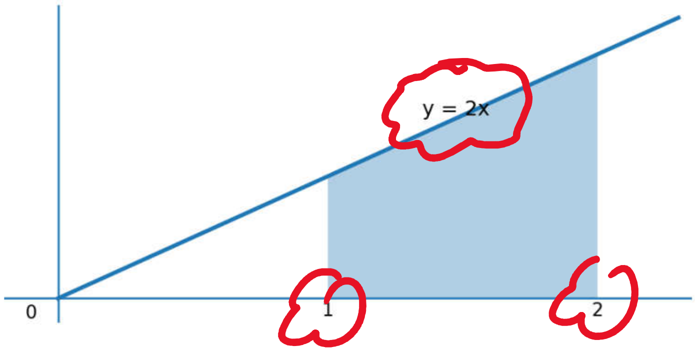
$= 2\sin\frac{x}{2} + 2\cos\frac{x}{2} + C$

$\int \sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}\, dx$

$2\cos\frac{x}{2} - 2\sin\frac{x}{2}$

$$-2\cos\frac{x}{2} - 2\sin\frac{x}{2}$$
$$= -2\left(\cos\frac{x}{2} + \sin\cdot\right)$$

Q26. The area of the shaded portion in the figure is _____.

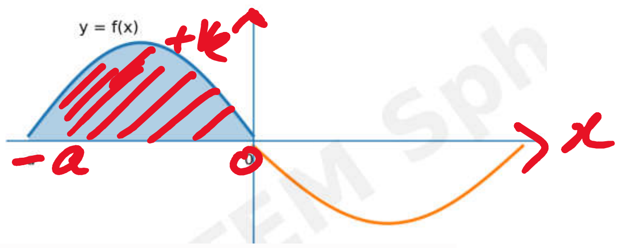


a) 3
b) 1/2
c) 1/9
d) 5x
e) 2x

$$\int_1^2 2x\,dx = 2\left|\frac{x^2}{2}\right|_1^2$$
$$= 4 - 1 = 3$$

Q27. The area of the shaded region in the given figure is equal to _____.

- a) $2\int_0^a f(x)\,dx$
- b) $(1/2)\int_0^a f(x)\,dx$
- c) $-\int_{-a}^0 f(x)\,dx$
- d) $\int_{-a}^0 f(x)\,dx$ ✓
- e) $2\int f(x)\,dx$



Q28. ∫ sec²x dx = ?

a) tan x ✓
b) sin x
c) cot x
d) sec x
e) cosec x

$$\frac{d}{dx}(\tan x) = \int \sec^2 x\,dx$$

Q29. The value of $\int_0^2 4e^{2t}\,dt$ is:

- a) $8e^4$
- b) $2e^4$
- c) $2(e^4-1)$
- d) e^4
- e) $2(e^4+1)$

$$4 \cdot \frac{\left. e^{2t} \right|_0^2}{\cancel{2}}$$

$$= \boxed{2\left(e^4-1\right)}$$

Q30. ∫(sin x - cos x)² dx = ?

a) x + (sin 2x)/2
b) x + (cos 2x)/2 + C
c) x - (tan 2x)/2
d) x - (sec 2x)/2
e) x - (cos 2x)/2 + C

$$\underline{\sin^2 x + \cos^2 x} - \boxed{2\sin x\cos x}$$

$$\int \left(1 - \sin 2x\right) dx$$

$$= x + \frac{\cos 2x}{2} + C$$

Q31. $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x = ?$

- a) 4
- b) e^3
- c) e^5
- d) e^2

$$\lim_{x\to\infty}\left(1+\frac{1}{f(x)}\right)^{\frac{f(x)}{1}} = e$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{\frac{x}{3}\times 3}$$

$$= \left(\underline{\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{\frac{x}{3}}}\right)^3 = e^3$$

Q32. $\lim_{x\to 0}(1-a^{-x})(xa^{-x})^{-1} =$ ______.

- a) ∞
- b) 0
- c) $\log_a e$
- d) 1
- e) $\log_e a$

$$\lim_{x\to 0} \frac{1-a^{-x}}{xa^{-x}} \times \frac{a^x}{a^x}$$

$$= \lim_{x\to 0} \frac{a^x-1}{x} = \ln a$$

$$= \log_e a$$

Q33. $\lim_{x\to\infty} \frac{x^3-3x^2+2x+1}{3x^3-4x^2+2x+5} =?$

- a) $1/3$
- b) $1/9$
- c) $-2/9$
- d) $-2/3$

$$\frac{1x^3}{3x^3}$$

$$\frac{x^4 - \dots}{3x^3 - \dots}, \infty$$

$$\frac{x^3 - \dots}{4x^3 - \dots}, 0$$

Q34. $\lim_{x\to 0} \frac{\sin^2 x - 1}{\sin^2 x - 2\sin x + \cos^2 x} =$ ______.

- a) $\sin x + 1$
- b) 1
- c) $\sin x - 1$

$$\sin^2 x + \cos^2 x = 1$$

$$= -\cos^2 x$$

$$\frac{0-1}{1-0} = \frac{-1}{1}$$

sin

- c) $\sin x - 1$
- d) -1
- e) 0

$1-0$ $\boxed{1}$ sin

$$\lim_{x\to 0} \frac{1-\cos x}{x} = \left(\frac{0}{0}\right) \text{fr}$$

L' Hospital $\Rightarrow \lim_{x\to 0} \frac{+\sin x}{1} = \frac{0}{1} = 0$

$k = 0, 1, 2, \cdots, n$

Q35. If $S_n = \sum_{k=0}^{n} ar^k$ and $0 < r < 1$, then $S_\infty = ?$

a) $a/(1-r)$
b) $(1-r)/a$
c) $1/(1-r)$
d) $(1+r)/a$
e) $a/(1+r)$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

$$S_n = ar^0 + ar^1 + ar^2 + \cdots + ar^n$$

$$= a + ar + ar^2 \cdots$$

$$r = \frac{ar}{a}$$

Q36. Find four numbers in A.P. whose sum is 24 and sum of squares is 164.

a) 3, 6, 8, 7
b) 3, 5, 7, 9
c) 4, 3, 2, 9
d) -3, 5, -6, 4
e) -4, -3, -2, -1

$9+36+64+49$

$9+25+49+81$

α e) -4, -3, -2, -1 $\rightarrow 9+25+49+81$

Q37. If $^nC_5 = {}^nC_3$, then n = $\underline{5}+3=8$.

a) 2
b) 8
c) 10
d) 5
e) 15

$q =$ denominator

Q38. If $y = \underbrace{\frac{1}{3}}_{q} + \frac{1\cdot3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1\cdot3\cdot5}{3!}\left(\frac{1}{3}\right)^3 + \ldots$, then $y^2 + 2y =$ _____.

a) 0
b) 1
c) 2
d) 3

$$(1+x)^n = 1 + nx + \frac{n(n+1)}{2}x^2 + \cdots$$

$$1+y = 1 + \frac{1}{3} + \cdots$$

$$\boxed{1+y = \sqrt{\frac{q}{q-2}}} = \sqrt{\frac{3}{1}}$$

$$1+y = \sqrt{3}$$

$$1+y^2+2y = 3$$

$$y^2+2y = 2$$

Q39. If the H.M. between 3 and x is 3, then x = _____.

a b

Q39. If the H.M. between 3 and x is 3, then x = _____.

a) 1/3

b) 9/2

c) 2/3

d) 0

e) 3

$$H = \frac{2ab}{a+b}$$

$$3 = \frac{2 \times 3x}{3+x}$$

$$9 + 3x = 6x \Rightarrow 3x = 9$$

$$x = 3$$

Q40. 1 + 2 + 3 + … + n = _____. S_n

a) 2n + 1

b) n(n+1)(2n+1)/6

c) n(n+1)/2

d) [n(n+1)/6]²

e) n²

$n = 1$ $\quad S_1 = 1$

$n = 2$ $\quad S_2 = 3$

$$\frac{1(2)(3)}{6}$$

$$\frac{1(2)}{2}$$

$$\frac{1(2)}{6}$$

$$\frac{2(3)}{2}$$

Q41. Which term in the expansion of (x/2 + 2/x²)¹² is independent of x?

a) 6

b) 5

c) 13

d) 1

x^0

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$= 12(1)(9)$

$\left(ax^p+\frac{1}{bx^q}\right)^n$ $r = \frac{np}{p+q}$, $= \frac{12(1)}{3} = 4$

Algebra and Complex Numbers

Q42. The sum of five different positive odd integers each less than 16 is equal to the square of an integer x. The least value of x is _____.

a) 6
b) 5 ✓
c) 7
d) 8
e) 3

1, 3, 5, 7, 9, 11, 13, 15

25 = $(5)^2$

$1^2=1$

$2^2=4$ $3^2=9$, $4^2=16$, $5^2=25$

Q43. The positive values of x for which $9x^{2/3}+4x^{-2/3}=37$ are:

a) $(1/27, 8)$
b) $(27, 8)$
c) $(64, 27)$
d) $(1, 125)$
e) $(81, 9)$

$9\cdot(2^3)^{\frac{2}{3}} + 4\cdot(2^3)^{-\frac{2}{3}}$

$9\times4 + 4\cdot\left(\frac{1}{4}\right)$

36+1

$9(3^3)^{\frac{2}{3}}$

$9(9)+4(\ )$

$9(3^{-3})^{\frac{2}{3}} + 4(3^{-3})^{-\frac{2}{3}}$

$9\left(\frac{1}{9}\right) + 4(3^2)$

9+36

Q44. The multiplicative inverse of 1 - 2i is _____, where i = √-1.

a) -1 + 2i

Q44. The multiplicative inverse of 1 - 2i is _____, where i = √-1.

a) -1 + 2i

b) 2/5 + (1/5)i

c) 1/5 + (2/5)i

d) 3/5 + i

e) 1/4 + 5i

$1+4=5$

$$\frac{1}{5}+\frac{2i}{5}$$

$a+ib,\ a-ib$

$\{i, -i\}$

$=0$

Q45. If one root of x³ + 2x² + x + 2 is i, where i = √-1, which set contains all other solutions?

a) {-i, -2}

b) {2}

c) {-i}

d) {-i, 2}

e) {2, -2}

$-\cancel{8}+\cancel{8}-\cancel{2}+\cancel{2}=0$

$x^2(x+2)+1(x+2)=0$

$(x+2)(x^2+1)=0$

$x=-2$

$x^2=2^2=\pm 2$