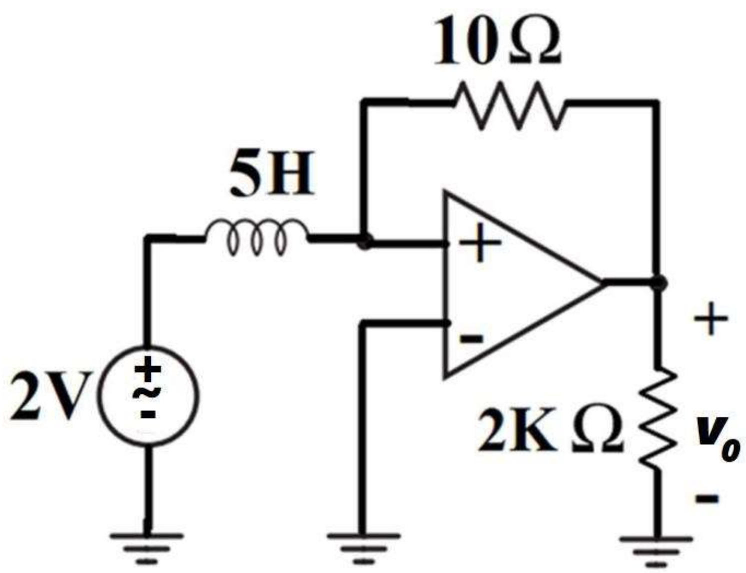
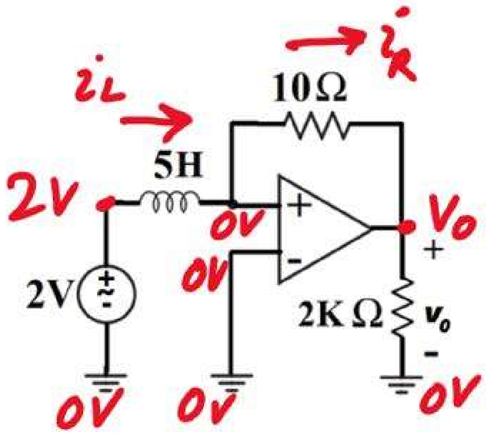
c) Assume ideal Op-amp characteristics. Given that $\boldsymbol{i_L(0^-) = 1\,A}$. Find $\boldsymbol{V_0\ at\ t = 1.4\ sec}$. Assume that voltage source stays at **2 v** for $\boldsymbol{0 \le t \le 2}$, and zero otherwise. **[5]**



Solution:



As the formula for current flowing through the inductor was given in the formula sheet provided

$$\boxed{i_L(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i_L(t_0)} \rightarrow (1) \quad t_0 = 0^-$$

And $i_L(0^-) = 1A$ (given)

But we know $i_R(t) = i_L(t)$

$$\Rightarrow \quad \frac{0 - V_o(t)}{10} = \frac{1}{5}\int_0^t 2\,dt + 1 \quad (\text{by } (1))$$

$$V_o(t) = -10\left[\frac{2}{5}t + 1\right] = -4t - 10$$

$$\boxed{V_o(t) = -4t - 10} \Rightarrow \boxed{V_o(1.4sec) = -15.6}$$

Mark Scheme Q4 part c:

Used inductor current equation from formula sheet -> 1-mark,

Correct working of KCL/Ohm's Law, integral function, limits and initial condition -> 3 mark

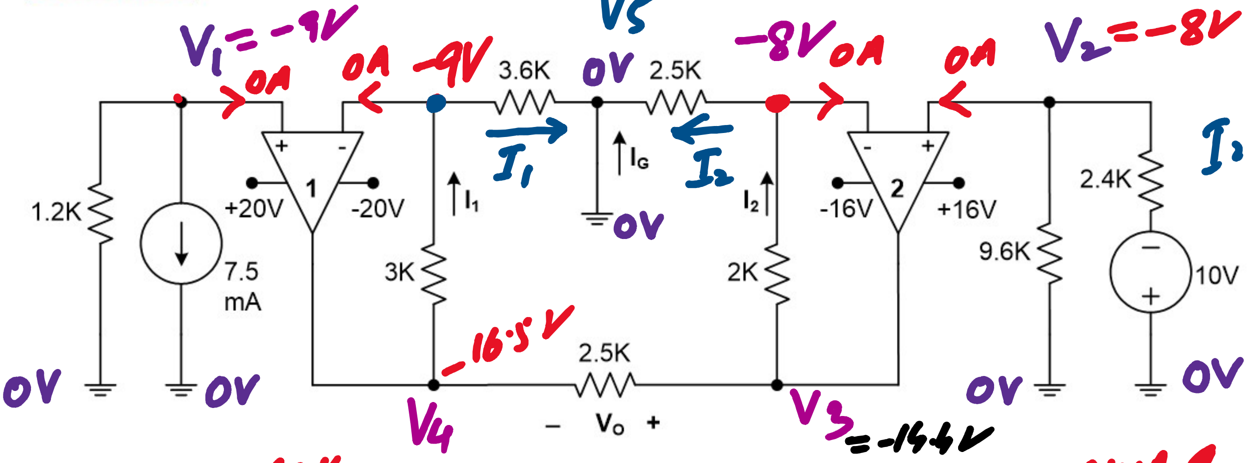
Seen -4t-10 or equivalent -> 1 mark

Op-Amp Problems

Wednesday, 18 March 2026 6:26 am

$V_o = V_3 - V_4 = +1V \Rightarrow v^+ = v^-$, $I^- = I^+ = 0A$

Question. Find I_1, I_2, I_G and V_O.



$I_2 = \frac{V_3 - (-8)}{2k}$

NVA @ Node V_1

$\frac{V_1 - 0}{1.2} + 7.5 = 0$

$\Rightarrow \boxed{V_1 = -9V}$

NVA @ V_2

$\frac{V_2 - 0}{9.6} + \frac{V_2 + 10}{2.4} = 0$

$\Rightarrow \boxed{V_2 = -8V}$

$I_1 = \frac{V_4 - (-9)}{3k}$

$I_1 = \checkmark$

NVA @ -8V

$\frac{-8 - V_3}{2} + \frac{-8 - 0}{2.5} = 0$

$\Rightarrow \boxed{V_3 = -14.4V}$

NVA @ -9V

$\frac{-9 - 0}{3.6} + \frac{-9 - V_4}{3} = 0$

$\Rightarrow \boxed{V_4 = -16.5V}$

KCL @ 0V (V_3)

$I_1 + I_2 + I_4 = 0$

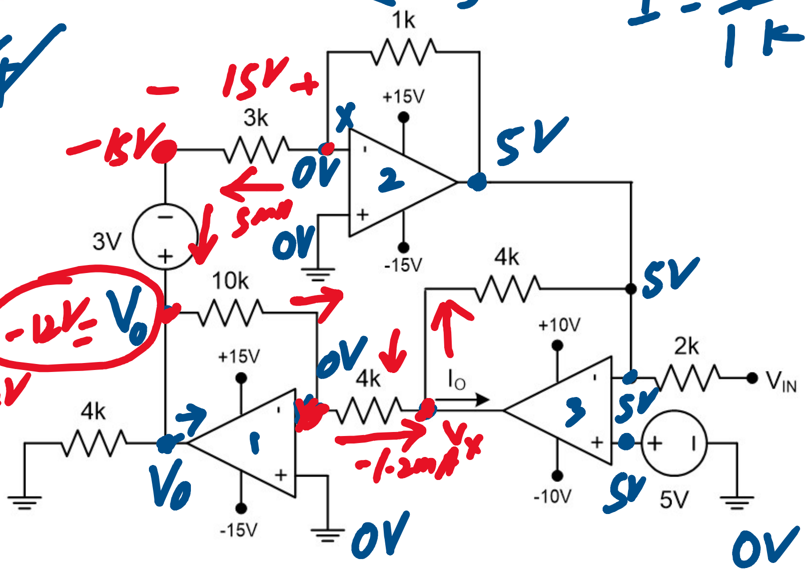
$\Rightarrow I_4 = -I_1 - I_2$

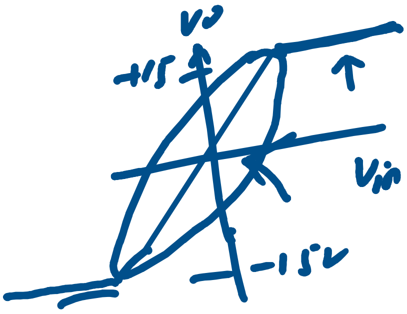
Question. Find I_O in terms of V_{IN}.

$\leftarrow 5mA$

$I = \frac{5-0}{1k}$

$V^+ = V^-$
$I^+ = I^- = 0A$





$V = (5)(3) = 15V$

$-15 + 3 = -12V$

$-12V = V_o$

V_x 3k 0V

$I_{10} = \frac{V_o - 0V}{10} \Rightarrow \boxed{I_{10} = -1.2mA}$

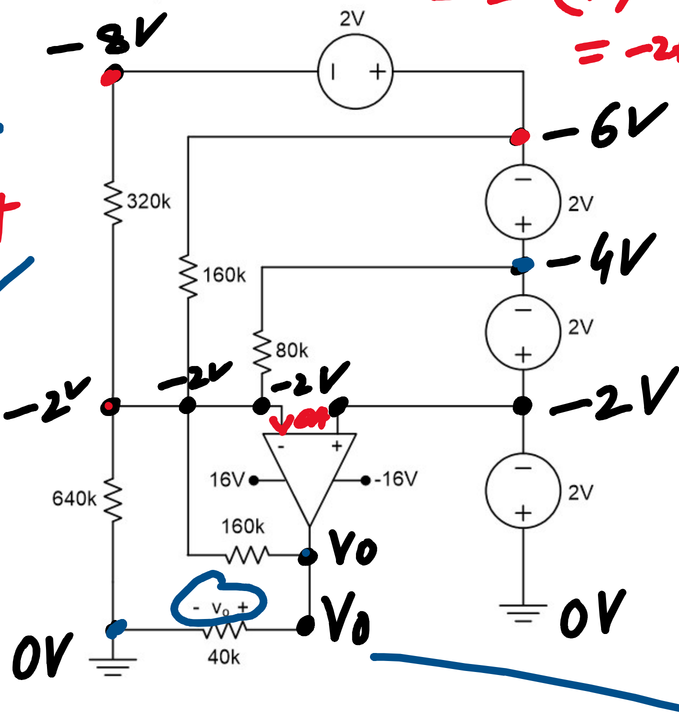
$0V - V_x = 15V$

$-1.2 = \frac{0 - V_x}{4} \Rightarrow \boxed{V_x = 4.8V}$

KCL @ V_x

$1.2 + I_o + \frac{4.8 - 5}{4} = 0 \Rightarrow I_o =$

Question. Find the value of V_o.



NVNG -2V

$$\frac{-2-(-8)}{320} + \frac{-2-(-6)}{160} + \frac{-2-(-4)}{80} +$$

$-2-(+2)-(-6)$

$= -2 + 6$

$0 - V_x = 2V$

$V^+ = V^-$

$I^+ = I^- = 0A$

$V_1 \quad R \quad V_2$

$$\frac{-2-0}{640} + \frac{-2-V_o}{160} = 0$$

$\Rightarrow V_o =$

$-\ V_o\ +$

0V $\qquad + V_1$

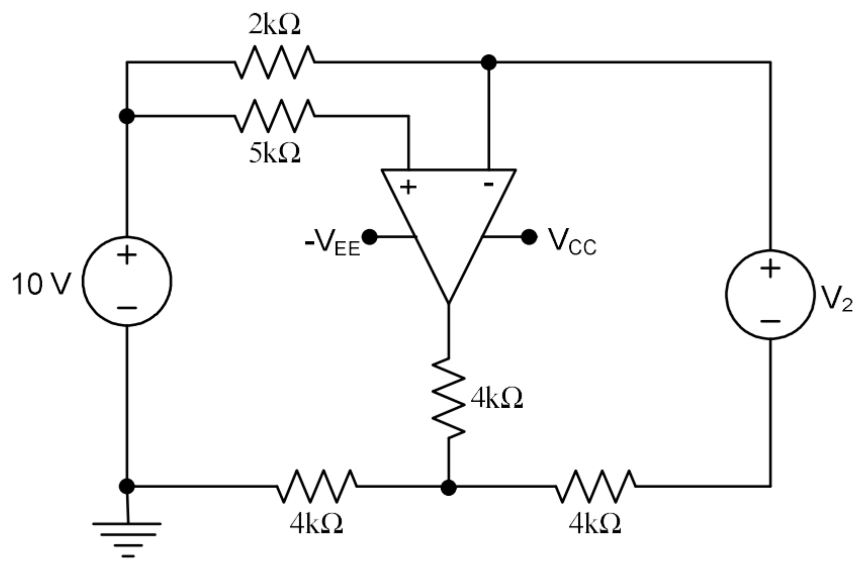
$$V_0 = V_1 - 0V \Rightarrow \boxed{V_1 = V_0}$$

Solution to the Most Important OpAmp , RC, RL & RLC Exam Oriented Problems

Q3: [**20 marks**] For the following circuit, **find** the **range of values of V_2** that will keep the op-amp operating in its linear region. Where,

$$V_{CC} = 20V, \qquad -V_{EE} = -20V$$

[Note that the output voltage of the op-amp should remain within $-V_{EE}\ to\ +V_{CC}$ voltage range if the op-amp has to be in its linear region.]
[**Marking scheme:** 50% marks for intermediary steps **only if** the end result is meaningful. 50% marks for correct end result.]



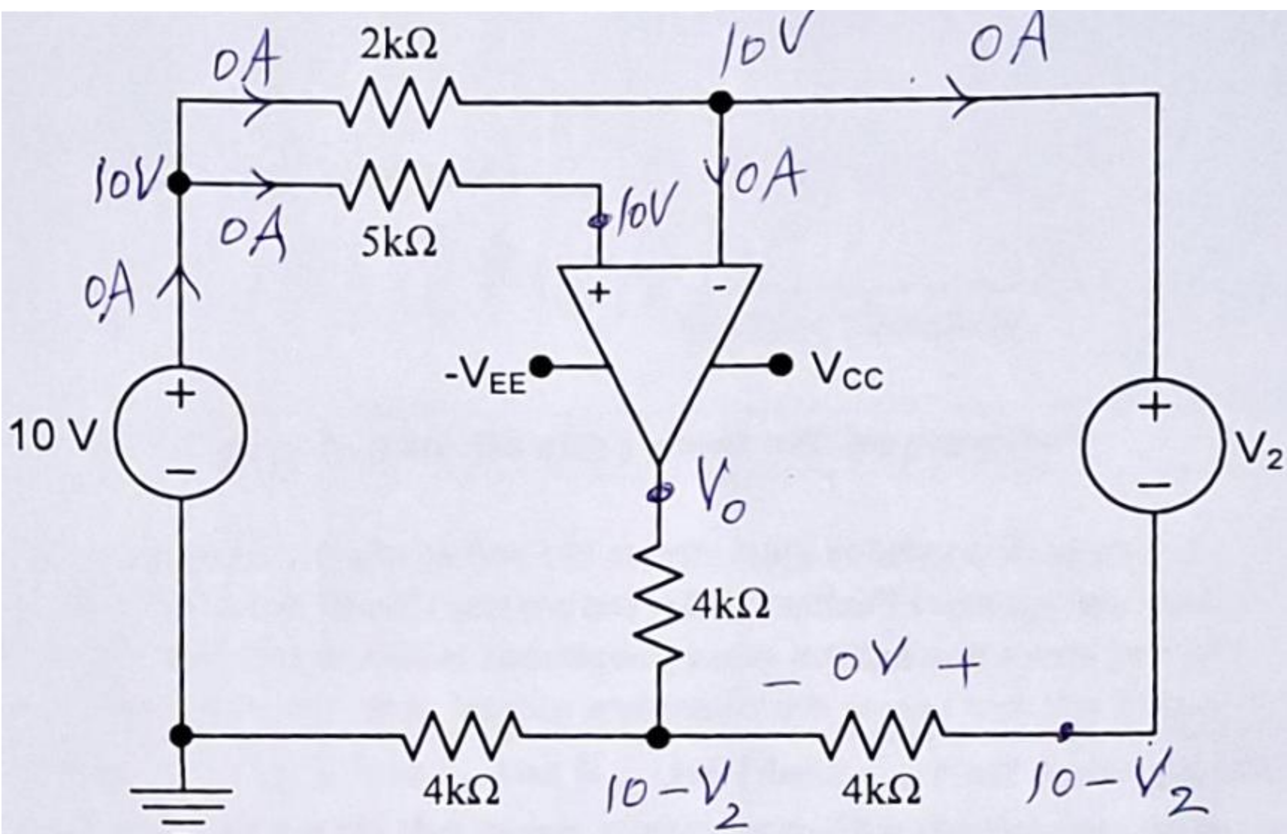


Fig. 2

For opamp to be in linear region, $-V_{EE} < V_o < V_{cc}$

V_o is as marked.

$$\frac{10 - V_2 - V_o}{4} + \frac{10 - V_2}{4} = 0$$

$$10 - V_2 - V_o + 10 - V_2 = 0$$

$$20 - 2V_2 - V_o = 0$$

$$\Rightarrow \quad V_o = 20 - 2V_2$$

For $V_o = -V_{EE}$: $\quad -V_{EE} = -20 = 20 - 2V_2 \Rightarrow V_2 = \frac{+40}{2}$

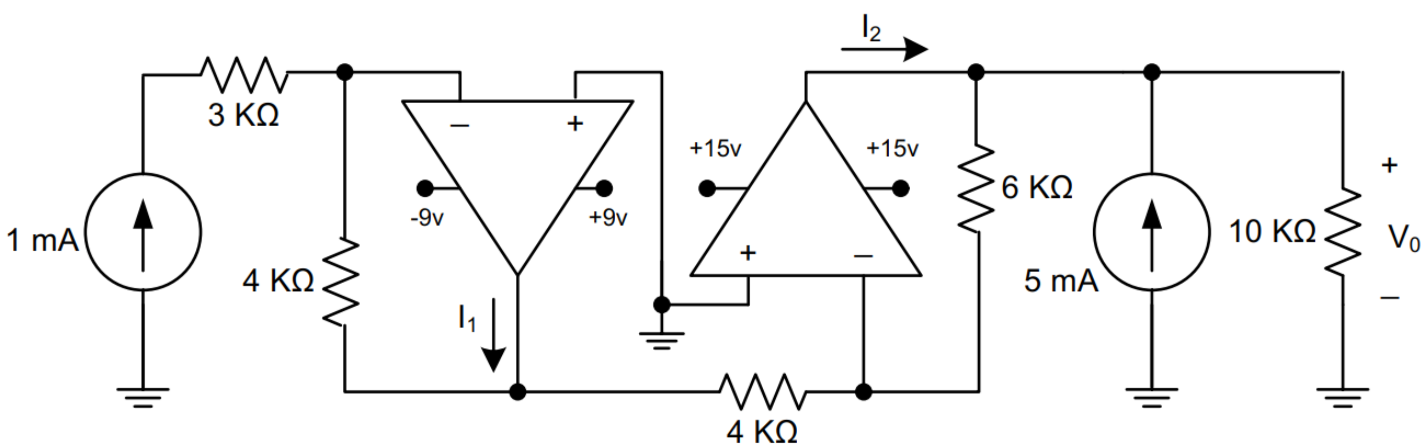
$V_2 = 20\,V$

For $V_o = V_{cc}$: $\quad V_{cc} = 20 = 20 - 2V_2 \Rightarrow V_2 = 0$

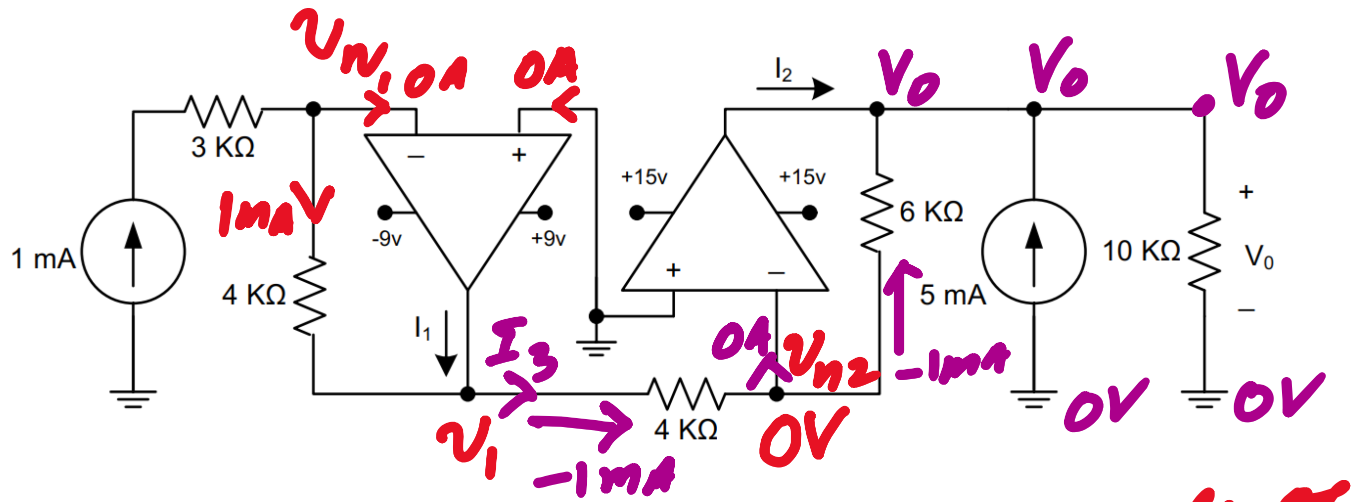
Thus to keep $-V_{EE} < V_o < V_{ce}$

$$\boxed{0 < V_2 < 20V}$$

Analyse the following cascaded OP-AMP circuit and determine v_0 , I_1 and I_2



b) Analyse the following cascaded OP-AMP circuit and determine v_o , I_1 and I_2.



As both non-inverting(+) terminals of both Op-Amps are grounded, so

$$v_n = v_p = 0V$$

NVA @ v_{n_1} (left op-amp)

$$-1mA + \frac{v_{n_1} - V_1}{4k} + 0 = 0 \Rightarrow v_{n_1} - v_1 = 4$$

$$\Rightarrow \boxed{v_1 = -4 \text{ Volts}} \quad (\because v_{n_1} = 0V)$$

NVA @ V_1

$$-1mA - I_1 + \frac{V_1 - 0}{4k} = 0$$

$$\Rightarrow \frac{-4}{4k} - 1mA = I_1$$

$$\Rightarrow \boxed{I_1 = -2mA}$$

Thus, at V_1 (KCL) $1mA - 2mA = I_3$

$$\Rightarrow \boxed{I_3 = -1mA}$$

$$\Rightarrow \frac{0 - V_0}{6k} = -1mA \Rightarrow \boxed{V_0 = 6V}$$

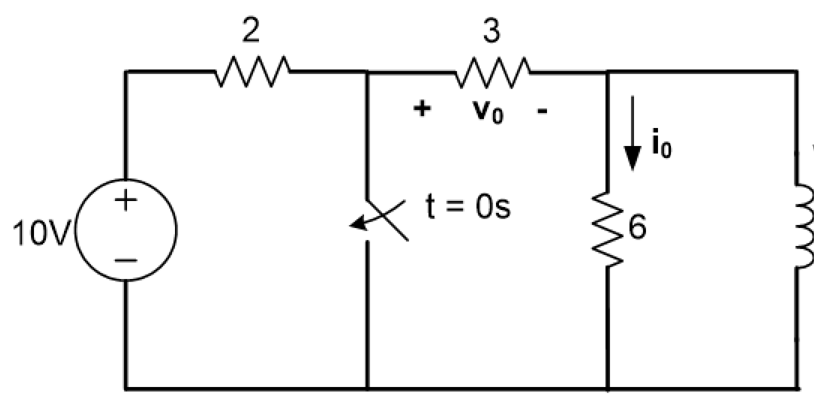
NVA @ V_0(6V) $-I_2 + 1mA - 5mA + \frac{6}{10k} = 0$

$$\Rightarrow \boxed{I_2 = -3.4mA}$$ Ans

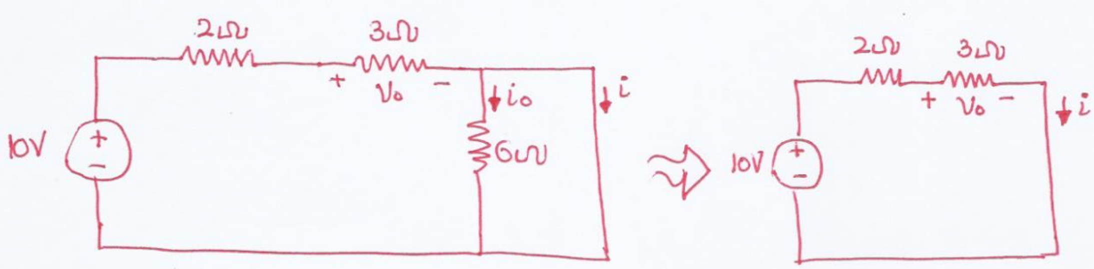
➢ **3 More Problems on Opamp have been covered at the end**

RC, RL , RLC and Opamp with Capacitor and Inductor

Assuming that the switch was open for a long time, find and plot $\mathbf{i_0}$, $\mathbf{v_0}$ and $\boldsymbol{i}$ for all time.



For $t<0$, Switch is OPEN for a long time (means Steady State has reached) Source is DC (not dependant on Time), So inductor behaves as Short cct meaning the 6Ω Resistor is Short-circuited. i.e.

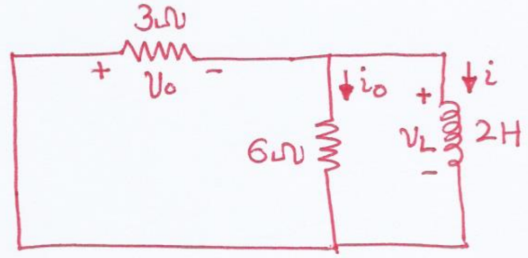


$\boxed{i_o = 0A}$

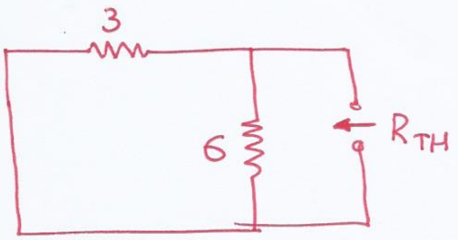
$i(t) = \dfrac{10}{2+3} \Rightarrow \boxed{i(t) = 2A}$ $\quad i(0-) = i(0+) = 2$

$V_o(t) = 3\,i(t) \Rightarrow \boxed{V_o(t) = 6V}$

For $t > 0$, Switch is CLOSED (means Voltage source is short-circuited)
Thus, we have a source-free RL cct i.e.



R_{TH} is Thevenin Resistance as seen by the energy storage element (i.e. inductor in this case) i.e



$\Rightarrow R_{TH} = 3 \| 6$

$\boxed{R_{TH} = 2\Omega}$

Thus $\tau = \frac{L}{R_{TH}} = \frac{2}{2} \Rightarrow \boxed{\tau = 1 sec}$

So, using response expression for source-free cct.

$i(t) = I_o e^{-t/\tau}$ $(t > 0)$

$\boxed{i(t) = 2e^{-t}}$ A $I_o = i(0) = i(0-) = 2A$

Since, inductor is in $\|$ with 6Ω and 3Ω resistors, so

$$v_o(t) = -v_L$$
$$= -L\frac{di}{dt}$$
$$= -2\frac{d}{dt}(2e^{-t})$$
$$= -2(-2e^{-t})$$

$\boxed{v_o(t) = 4e^{-t}}$ V

Finally, $i_o = \dfrac{v_L}{6}$

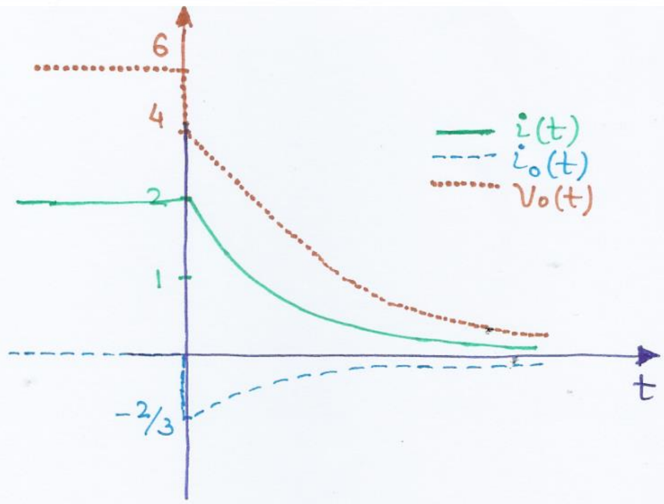
$$= \frac{-4e^{-t}}{6} \quad \text{or} \quad \boxed{i_o(t) = -\frac{2}{3}e^{-t}}\ \text{A}$$

Summarizing,

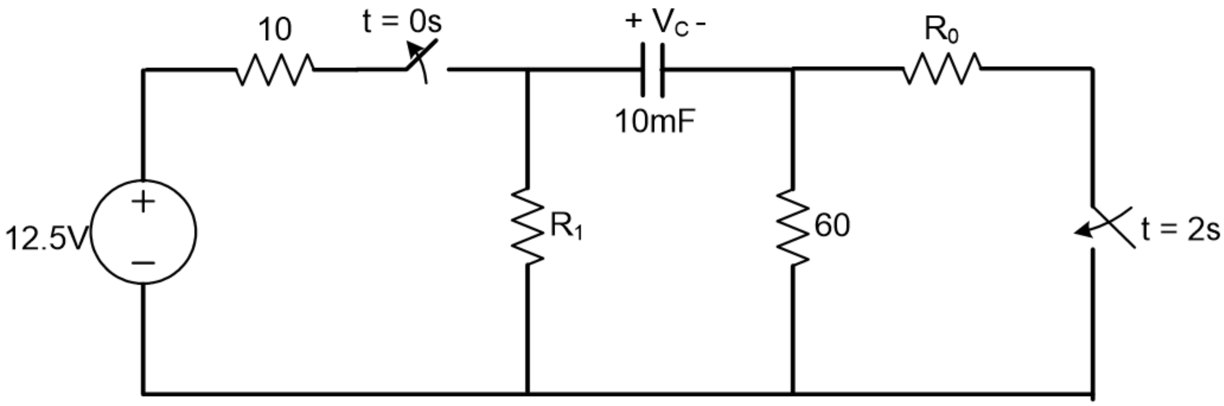
$$i_o(t) = \begin{cases} 0\ \text{A} & t < 0 \\ -\frac{2}{3}e^{-t}\ \text{A} & t > 0 \end{cases}$$

$$V_o(t) = \begin{cases} 6\ \text{V} & t < 0 \\ 4e^{-t}\ \text{V} & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2\ \text{A} & t < 0 \\ 2e^{-t}\ \text{A} & t > 0 \end{cases}$$

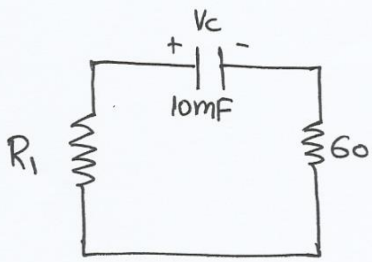


Select values for the resistors R_0 and R_1 in the network below such that $V_C(0.65) = 5.22V$ and $V_C(2.21) = 1V$. All Resistances are in Ω.



$V_c(0.65) = 5.22V$

means cct. for $t > 0$ (Source-free Circuit)



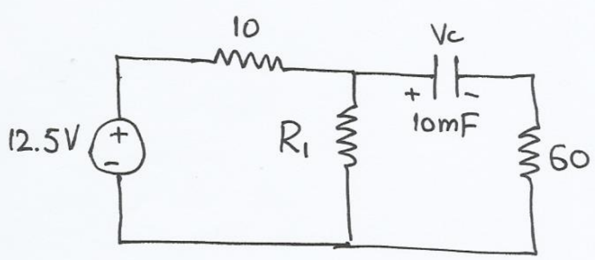
$$\Rightarrow V_c = V_0 e^{-t/\tau} = V(0) e^{-t/\tau}$$

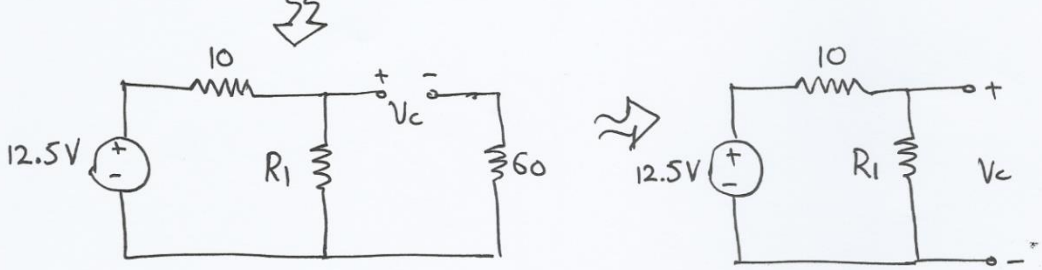
$$\tau = R_{Th} \cdot C = (R_1 + 60)(10\times10^{-3})$$

$$\boxed{\tau = (R_1+60)10m\,Sec}$$

$V_c(0^+) = V_c(0^-)$ = Can be found by cct. for $t < 0$

SS





(No Current flows through 60Ω)

$$v_c(0) = \frac{12.5\,R_1}{R_1+10}$$

$C = 10\,mF = 10\times10^{-3}$

So,
$$V_c = \frac{12.5\,R_1}{R_1+10}\, e^{\frac{-t}{(R_1+60)10^{-3}\times10}}$$

One Eq., one Unknown :)
Solve & Find R_1 Simply

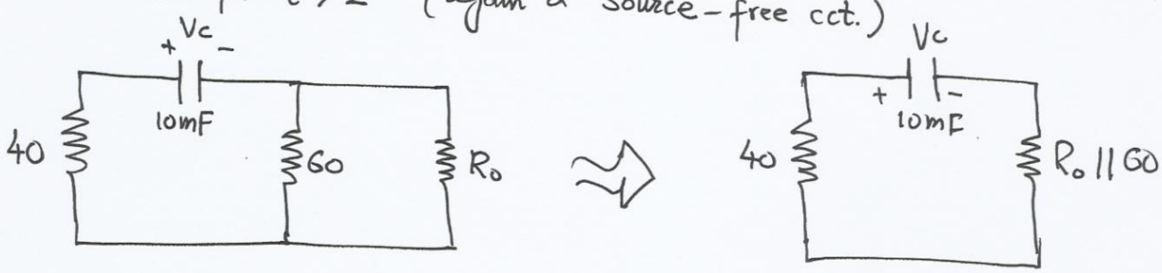
At $t = 0.65$ sec

$$5.22 = \frac{12.5\,R_1}{R_1+10}\, e^{-\frac{0.65}{(R_1+60)\times10^{-3}\times10}}$$

$$\boxed{R_1 = 40\,\Omega}$$

$V_c(2.21) = 1V$

means cct. for $t > 2$ (again a source-free cct.)



$$\tau = R_{TH}\cdot C$$
$$= \left[40 + \frac{60R_o}{R_o+60}\right](10\times10^{-3})\ \text{sec}$$

$$\boxed{\tau = 0.4 + \frac{0.6}{R_o+60}\ \text{Sec}} \quad \text{—}\ (A)$$

For $t > 2$

Caution: Please note this.

$$V_c(t) = V_c(2+)\, e^{-(t-2)/\tau} \quad \text{—— (1)}$$

$V_c(2+) = V_c(2-)$ can be found from eq. of previous interval with $R_1 = 40\,\Omega$

$$V_c(t) = \frac{12.5(40)}{40+10}\, e^{-\frac{t}{(40+60)(10\times10^{-3})}} \quad \text{(see last page)}$$

$$= 10\, e^{-t}$$

At $t = 2$,

$$V_c(2) = 10\, e^{-2} \Rightarrow \boxed{V_c(2) = 1.353\text{ V}}$$

From (1), at $t = 2.21$ sec

$$V_c(t) = V_c(2+)\, e^{-(t-2)/\tau}$$

$$1 = 1.353\, e^{-(2.21-2)/\tau}$$

$$0.74 = e^{\frac{-0.21}{\tau}}$$

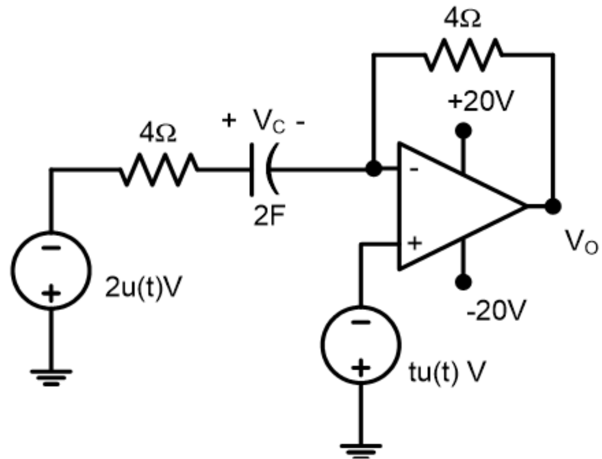
$$\Rightarrow \tau = -\frac{0.21}{\ln(0.74)} \qquad \boxed{\tau = 0.7\text{ sec}} \text{ — B}$$

Equating A and B $\quad 0.7 = 0.4 + \dfrac{0.6}{R_0 + 60}$

$$\Rightarrow \boxed{R_0 = 60\,\Omega}$$

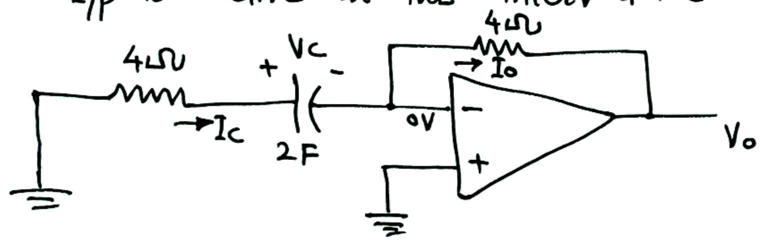
For the following circuit, find:

i. $V_O(0-)$
ii. $V_O(0+)$
iii. $V_O(\infty)$
iv. $V_O(t), t > 0$



i) $V_o(0-) = ?$

For $t<0$, no I/p is active in this interval i.e



$I_o = \frac{0-V_o}{4}$ Since $I_o = I_c = 0$

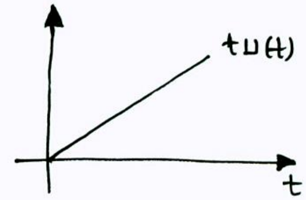
$V_c(0^-) = 0$ as well.

So, $0 = \frac{0-V_o}{4} \Rightarrow \boxed{V_o = 0V}$

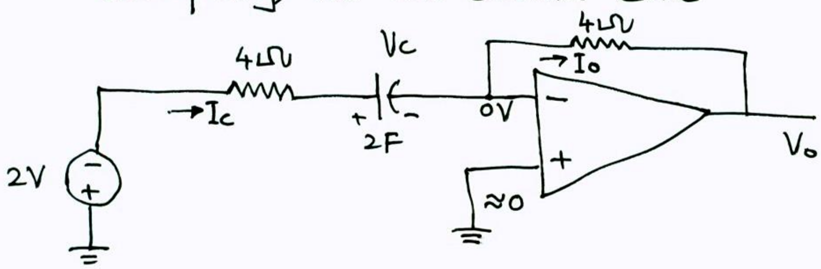
ii) $V_o(0+) = ?$

2V is active immediately at $t=0+$ but other I/p ($t\,U(t)$) is just about to start. So, for $t=0+$ we can safely say

$$t\,U(t) \approx 0$$



Consequently, the cct. becomes Like



$$V_C(0+) = V_C(0-) = 0$$

So, $I_c = \frac{-2-0}{4} = -\frac{1}{2}\,A$

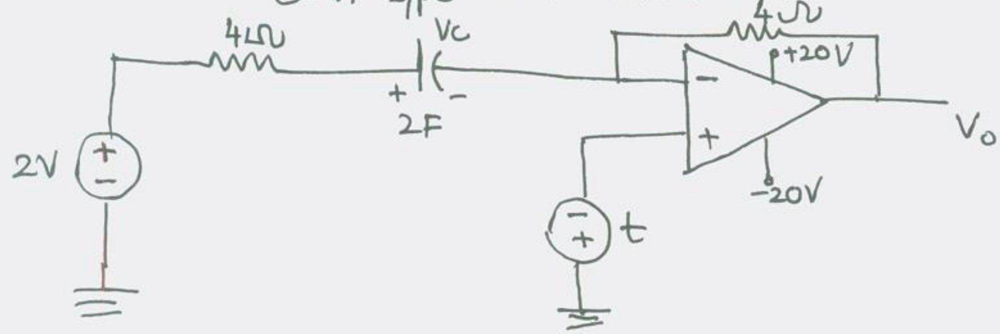
$$I_o = I_c = \frac{0 - V_o}{4} \Rightarrow V_o = -4\,I_c$$

$$= -4\left(-\tfrac{1}{2}\right)$$

$\boxed{V_o = 2\,V}$ at $t = 0+$

(iii) $V_o(\infty) = ?$ (steady state)

Both I/ps are active i.e.



Cap. will behave as open circuit for DC

$t = \infty$ means Infinite input voltage at non-inverting pin

⇓ Practically

Lead to Saturation of Op-amp. i.e. we can not get the O/p from Op-amp more than the Supply Voltage provided.

So, $\boxed{V_o(\infty) = 20V}$

iv) $V_o(t) \quad t > 0$

$$C \frac{dv_c}{dt} = \frac{V_a - V_o}{4}$$

$$4C \frac{dv_c}{dt} = V_a - V_o$$

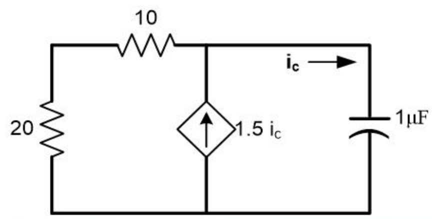
$$4C \frac{dv_c}{dt} = -t - V_o \qquad (V_a = -t)$$

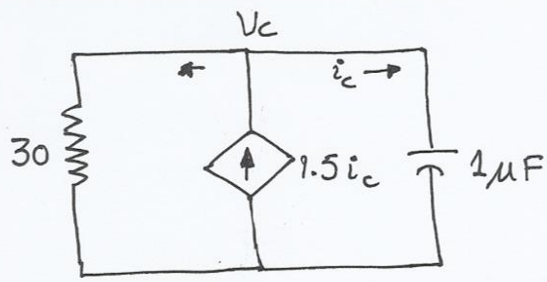
$$V_o = -t - 8 \frac{dv_c}{dt} \qquad (C = 2F)$$

or

$$\boxed{V_o(t) = -\left(t + 8\frac{dv_c}{dt}\right)}$$

The initial voltage across the capacitor in the circuit shown is non-zero but unknown. Determine the time when the voltage across the capacitor becomes half of its initial value. Assume the circuit to start operation at $t = 0$.





KCL at node labelled as 'V_c'

$$-\frac{V_c}{30} + 1.5\, i_c - i_c = 0$$

$$-\frac{V_c}{30} + 0.5\, i_c = 0$$

or $\quad V_c - 15\, i_c = 0 \qquad\qquad i_c = C\frac{d v_c}{dt}$

$$V_c - 15\times10^{-6}\frac{d}{dt}V_c = 0 \qquad\qquad = 1\times10^{-6}\frac{d}{dt}V_c$$

or

$$\frac{d}{dt}V_c - \frac{1}{15\times10^{-6}}V_c = 0$$

Solving Diff. Equation

$$V_c(t) = V_0\, e^{\frac{t}{15\times10^{-6}}} \quad V$$

Time when $V_c(t) = \frac{1}{2} V_o$ is thus

$$\frac{V_o}{2} = V_o \, e^{\frac{t}{15\times10^{-6}}}$$

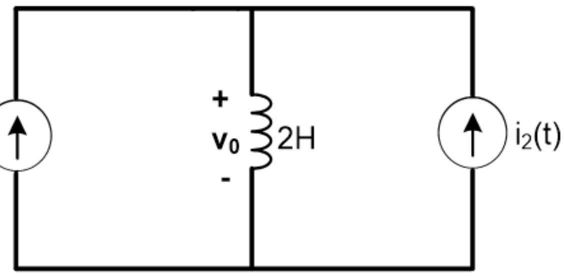
$$\Rightarrow \quad \frac{1}{2} = e^{\frac{t}{15\times10^{-6}}}$$

$$\ln(0.5) = \ln e^{\frac{t}{15\times10^{-6}}}$$

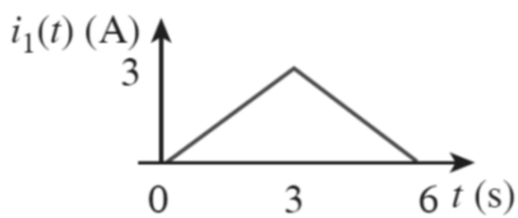
$$\Rightarrow \quad t = 15\ln(0.5) \ \mu\text{Sec}$$

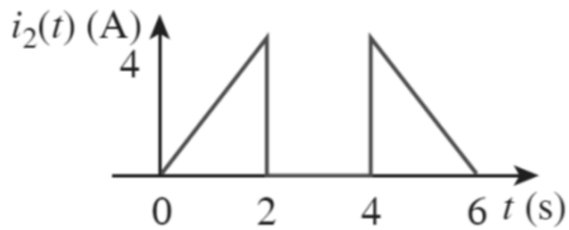
$$\boxed{t = -10.4 \ \mu\text{Sec}}$$

Sketch V_0 for the circuit given below:



where

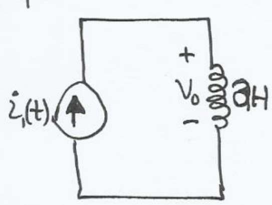




There are two I/p Sources $i_1(t)$ and $i_2(t)$, So applying Superposition Theorem

$V_o = V_1 + V_2$ $\quad\begin{cases} V_1 - \text{response due to } i_1 \\ V_2 - \text{" " " } i_2 \end{cases}$

V_1 :



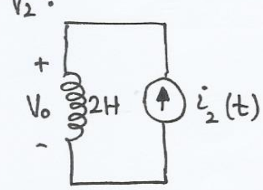
Remember i_1 & i_2 are NOT DC !

$$V_1 = L\frac{di_1}{dt}$$

$$= 2\frac{di_1}{dt} \quad \text{where } i_1 = \begin{cases} t & 0<t<3 \\ -t & 3<t<6 \end{cases}$$

So, $\boxed{V_1 = \begin{cases} 2 & 0<t<3 \\ -2 & 3<t<6 \end{cases}}$

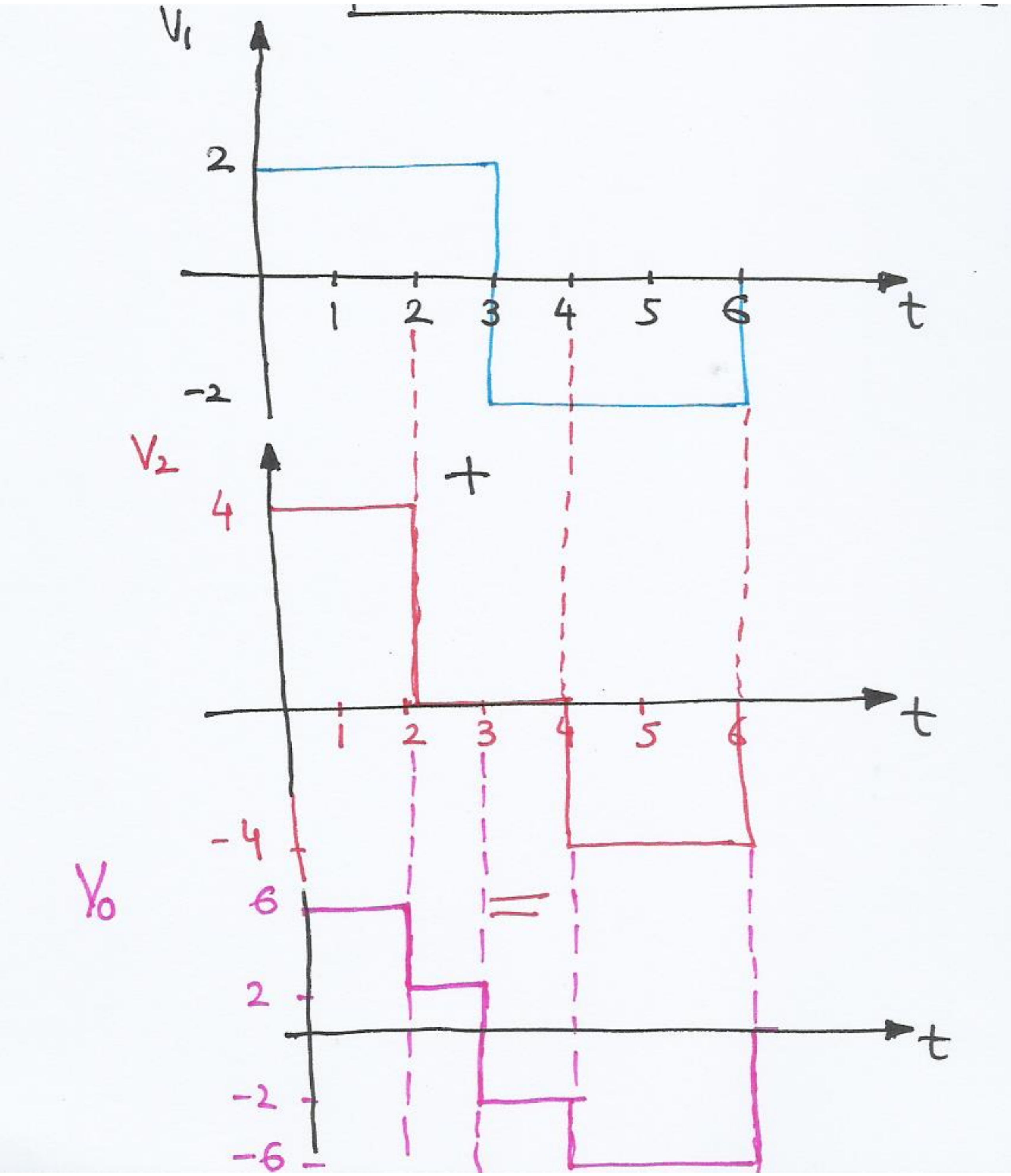
V_2 :



$$V_2 = L\frac{di_2}{dt}$$

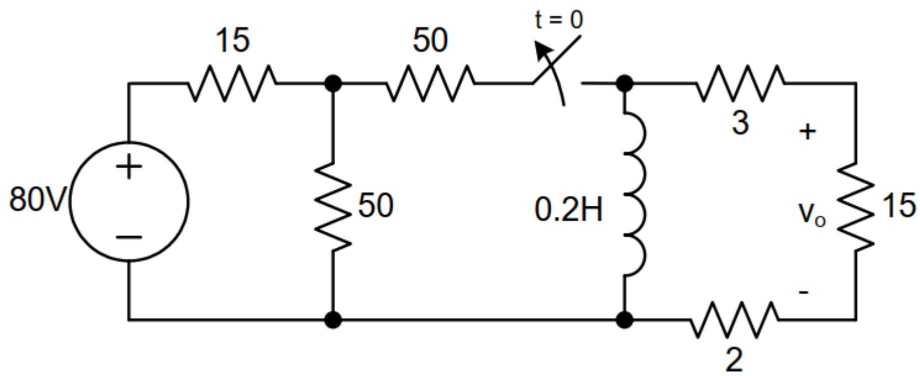
$$= 2\frac{di_2}{dt} \quad \text{where } i_2 = \begin{cases} 4t & 0<t<2 \\ 0 & 2<t<4 \\ -4t & 4<t<6 \end{cases}$$

So, $\boxed{V_2 = \begin{cases} 4 & 0<t<2 \\ 0 & 2<t<4 \\ -4 & 4<t<6 \end{cases}}$



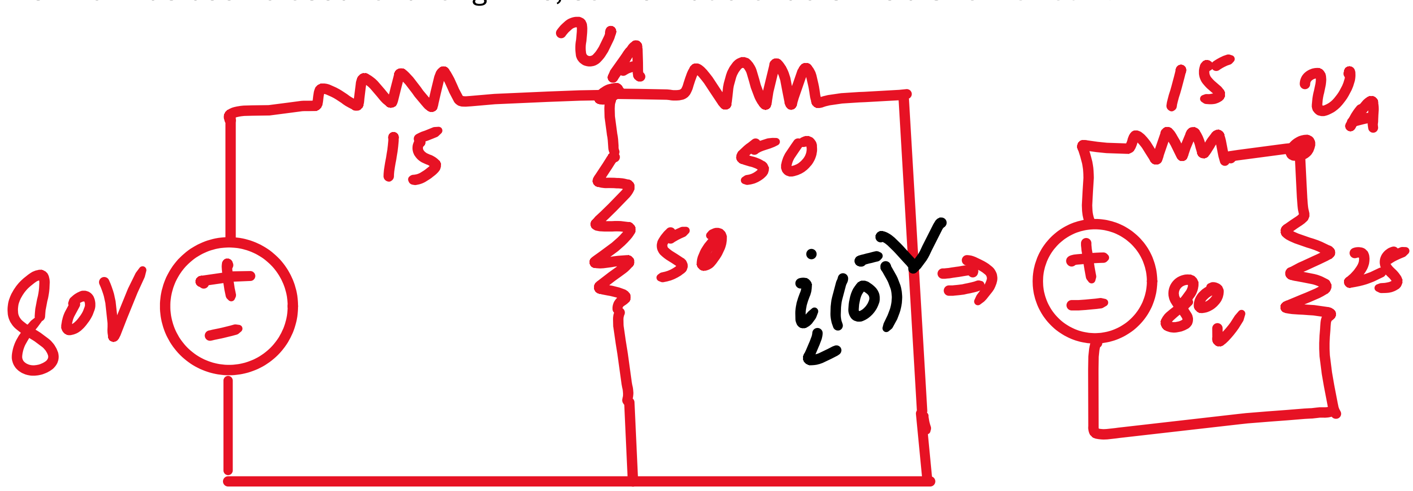
V1
2
-2
1 2 3 4 5 6
t
V2
4
+
1 2 3 4 5 6
t
-4
Vo
6
=
2
-2
-6
t

The switch in the circuit in figure below has been closed for a long time. At t = 0 it is opened. Find Vo(t) for t > 0.



1. Before opening switch: $t = 0^-$

Switch has been closed for a long time, so the inductor acts like a **short circuit**.



Right side has no source, so no current flows in 3Ω, 15Ω, 2Ω are shorted out.

On left side, the node after 15Ω sees:

$$50\Omega \parallel 50\Omega = 25\Omega$$

So, node voltage is:

$$V_A = 80\left(\frac{25}{15+25}\right) = 80\left(\frac{25}{40}\right) = 50V$$

Current through the top 50Ω resistor is:

$$i_L(0^-) = \frac{50}{50} = 1A$$

So,

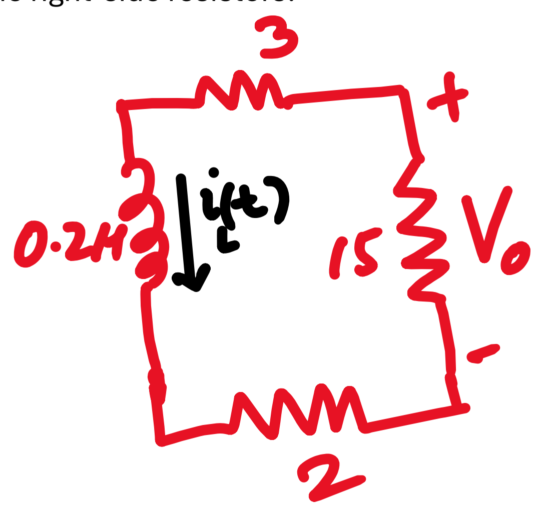
$$i_L(0^+) = i_L(0^-) = 1A$$

Inductor current cannot change instantly.

2. After opening switch: $t > 0$

Left side is disconnected. The inductor discharges through the right-side resistors:

$$R_{\text{eq}} = 3 + 15 + 2 = 20\Omega$$



Time constant:

$$\tau = \frac{L}{R_{eq}} = \frac{0.2}{20} = 0.01s$$

So, inductor current is:

$$i_L(t) = I_L(0)e^{-t/\tau}$$
$$i_L(t) = e^{-100t}A$$

3. Find $v_o(t)$

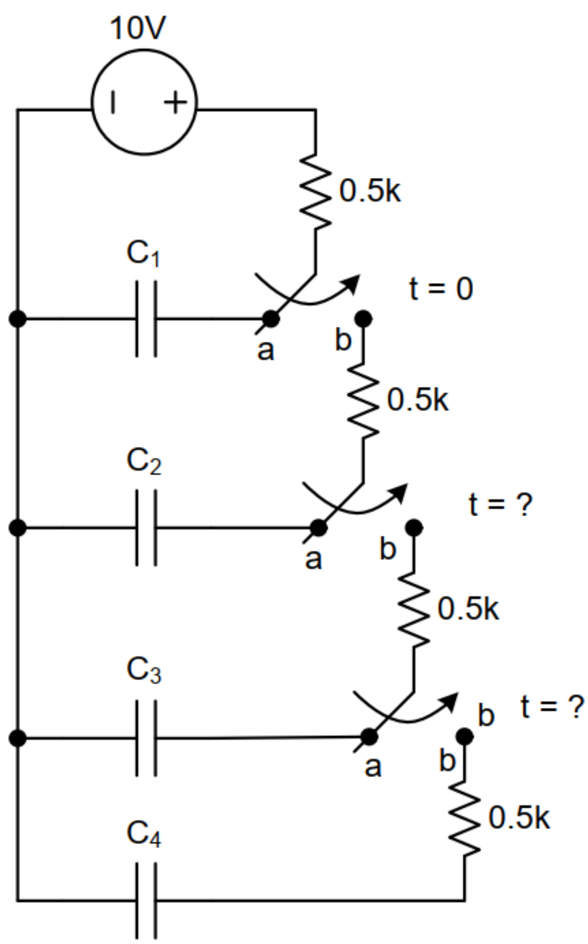
The inductor current is downward, so the resistor-loop current flows opposite direction. Therefore, current through the 15Ωresistor is upward, opposite to the marked $+$to $-$polarity.

So,

$$v_o(t) = -I_L(t)R = -(15)(e^{-100t})$$
$$\boxed{v_o(t) = -15e^{-100t}\text{ V}}$$

Negative sign means the bottom of the 15Ωresistor is actually at higher potential than the top.

Each capacitor (C_i) in the following circuit is of **10mF** value and initially uncharged. The first switch (S_1) moves from position **a** to position **b** at **t = 0** (as shown) disconnecting C_1. S_2 operates and disconnects C_2 when it has charged to **6.32V** and S_3 operates and disconnects C_3 when it has charged to **6.32V**. C_4 is then charged to maximum value. Find the **total time**, in **seconds**, for C_4 to charge to a **voltage greater than 9.9V**.



Each capacitor:

$$C = 10\text{ mF} = 0.01F$$

Each resistor:

$$R = 0.5k\Omega = 500\Omega$$

Supply voltage:

$$V_s = 10V$$

For capacitor charging:

$$V_C = V_s\left(1 - e^{-t/RC}\right)$$

Important point

$$6.32V = 63.2\% \text{ of } 10V$$

And a capacitor reaches 63.2%of final voltage in **one time constant**.

So, when capacitor voltage reaches $6.32V$,

$$t = \tau = RC$$

Step 1: Time for C_2

After S_1moves to b, C_2charges through two resistors:

$$R = 0.5k + 0.5k = 1k\Omega$$
$$\tau_2 = RC = (1000)(0.01) = 10s$$

So,

$$t_2 = 10s$$

Step 2: Time for C_3

Now C_3charges through three resistors:

$$R = 0.5k + 0.5k + 0.5k = 1.5k\Omega$$
$$\tau_3 = RC = (1500)(0.01) = 15s$$

So,

$$t_3 = 15s$$

Step 3: Time for C_4 to become greater than $9.9V$

Now C_4 charges through four resistors:

$$R = 0.5k + 0.5k + 0.5k + 0.5k = 2k\Omega$$
$$\tau_4 = RC = (2000)(0.01) = 20s$$

For C_4:

$$9.9 = 10\left(1 - e^{-t/20}\right)$$

Divide by 10:

$$0.99 = 1 - e^{-t/20}$$
$$e^{-t/20} = 0.01$$

Taking natural log:

$$-\frac{t}{20} = \ln(0.01)$$
$$t = 20\ln(100)$$
$$t = 92.1s$$

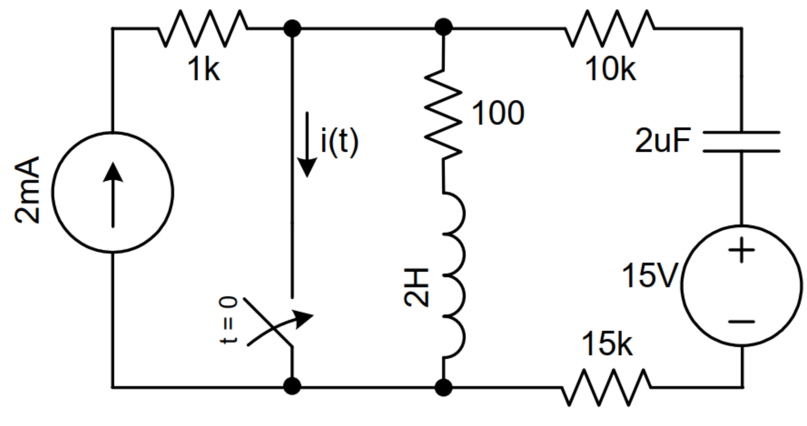
Total time after $t = 0$

$$T = t_2 + t_3 + t_4$$
$$T = 10 + 15 + 92.1$$
$$\boxed{T = 117.1s}$$

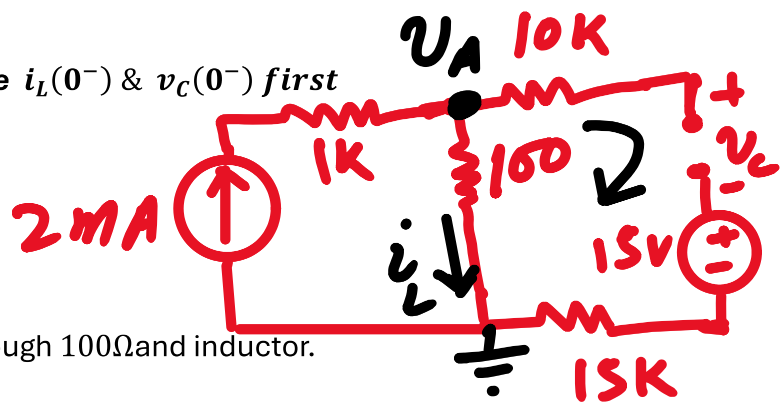
If the question wants time starting from the very beginning when C_1 was also initially uncharged, then add time for C_1:

$$\tau_1 = (500)(0.01) = 5s$$
$$T = 5 + 10 + 15 + 92.1$$
$$\boxed{T = 122.1s}$$

The switch in the circuit below is closed at **t = 0**. Find **i(t)** for **t ≥ 0.**



1. Before switch closes: $\boldsymbol{At\ t = 0^-}$ ***(Compute*** $\boldsymbol{i_L(0^-)\ \&\ v_C(0^-)\ first}$



Switch is open for a long time, so:

- Inductor acts as short circuit.
- Capacitor acts as open circuit.

The $2mA$ current flows through $1k\Omega$, then through $100\Omega$and inductor.

So,

$$i_L(0^-) = 2mA$$

Therefore,

$$i_L(0^+) = 2mA$$

Voltage across 100Ω:

$$V_A = IR = (2mA)(100) = 0.2V$$

For capacitor: $v_C(0^-)$, Apply KVL in the RHS Open Loop (Current = 0A due to Open Loop)

$$-V_A + v_C(0^-) + 15 = 0$$

Hence,

$$v_C(0^-) = 0.2 - 15 = -14.8V$$

So,

$$v_C(0^+) = -14.8V$$

Step 2: After switch closes, $t \geq 0$

The switch shorts the top and bottom nodes.

The switch current $i(t)$has three contributions: (Apply KCL at Top Node)

$$-2mA + i(t) + i_L(t) + i_C(t) = 0$$

$\boldsymbol{i(t) = 2mA - i_L(t) - i_C(t)}$**--------------------------------------→ (1)**

Inductor branch

The inductor discharges through 100Ω.

$$\tau_L = \frac{L}{R} = \frac{2}{100} = 0.02s$$
$$i_L(t) = 2mAe^{-t/0.02}$$
$$i_L(t) = 2mAe^{-50t}$$

Capacitor branch

For the right branch:

$$R = 10k + 15k = 25k\Omega$$
$$\tau_C = RC = (25k)(2\mu F) = 0.05s$$

Final capacitor voltage:

$$v_C(\infty) = -15V$$

As

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau_C}} = -15 + (-14.8 - (-15))e^{-20t} = -15 + 0.2e^{-20t}$$

So:

$$v_C(t) = -15 + 0.2e^{-20t}$$

Capacitor current:

$$i_C = C\frac{dv_C}{dt}$$
$$i_C = (2\mu F)(-4e^{-20t})$$
$$i_C = -8\mu Ae^{-20t}$$

Negative sign means actual current enters the top node from the right branch.

Step 3: Thus Eqn. (1) becomes

$$i(t) = 2mA - i_L(t) - i_C(t)$$
$$i(t) = 2mA - 2mAe^{-50t} - (-8\mu Ae^{-20t})$$
$$\boxed{i(t) = 2mA - 2mAe^{-50t} + 8\mu Ae^{-20t}}$$

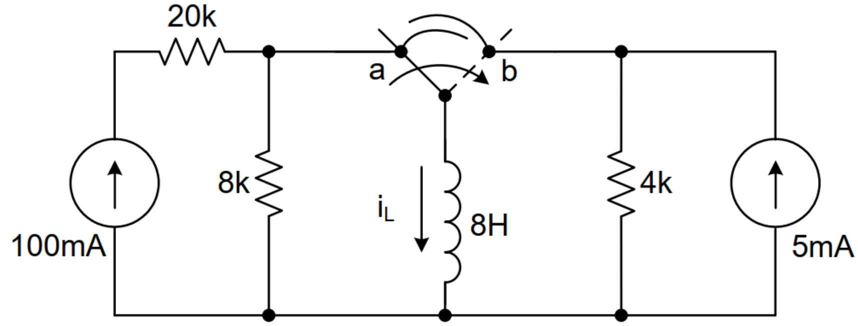
Final answer:

$$\boxed{i(t) = 2mA - 2mAe^{-50t} + 8\mu Ae^{-20t}}, t > 0$$

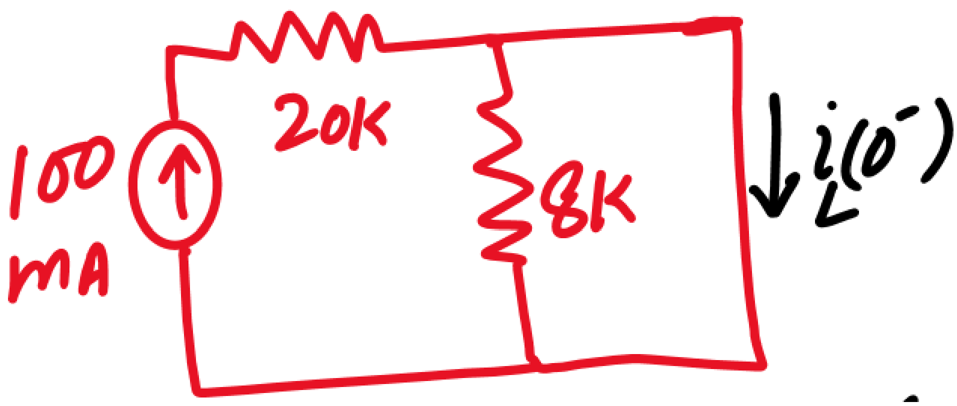
For the circuits given below, find the specified current/voltage just before switching and then at t = ∞.

$i_L(0^-)$ = ______A

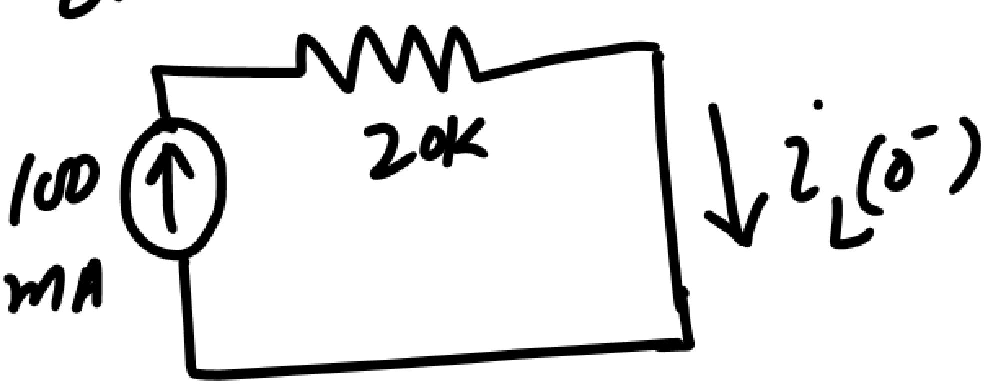
$i_L(\infty)$ = ______A



At $t=0^-$, inductor will be in steady state i-e, short cct.



8k will be shorted out

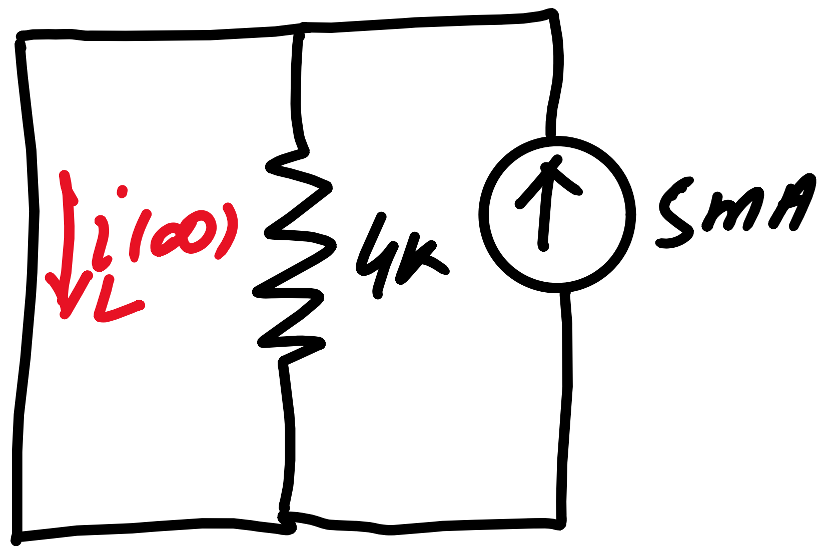


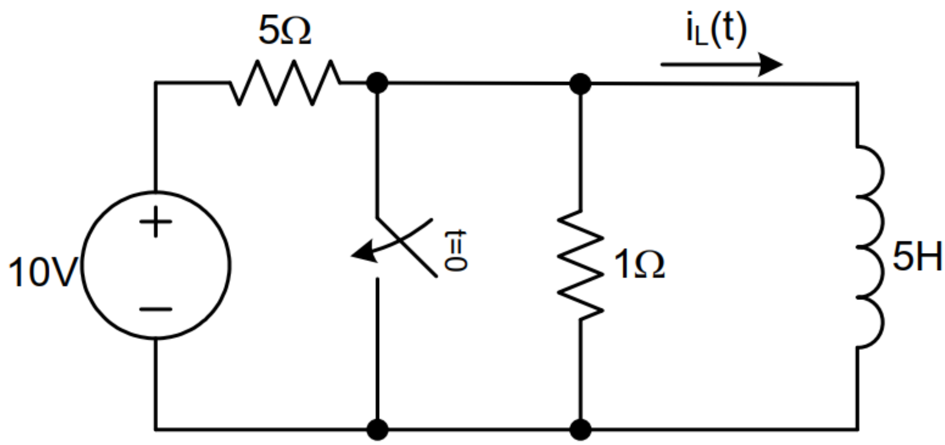
$\Rightarrow$ $\boxed{i_L(0^-) = i_L(0^+) = 100\,mA}$

At $\underline{t=\infty}$, inductor will be short cct.

$\Rightarrow$ 4k will be shorted out

$\boxed{i_L(\infty) = 5\,mA}$ Ans





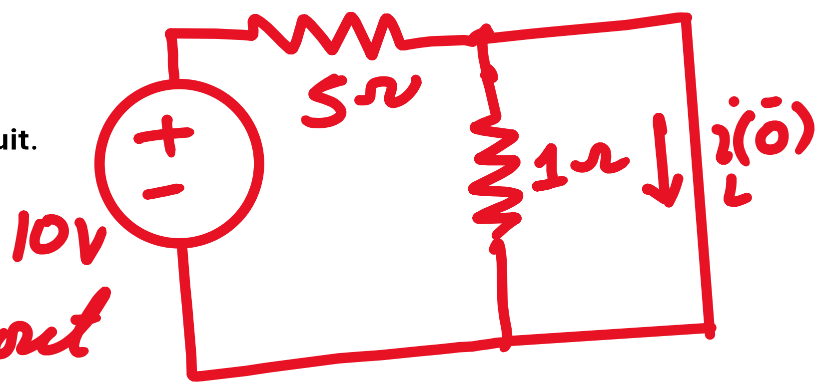
$i_L(0^-)$ = ______A

$i_L(\infty)$ = ______A

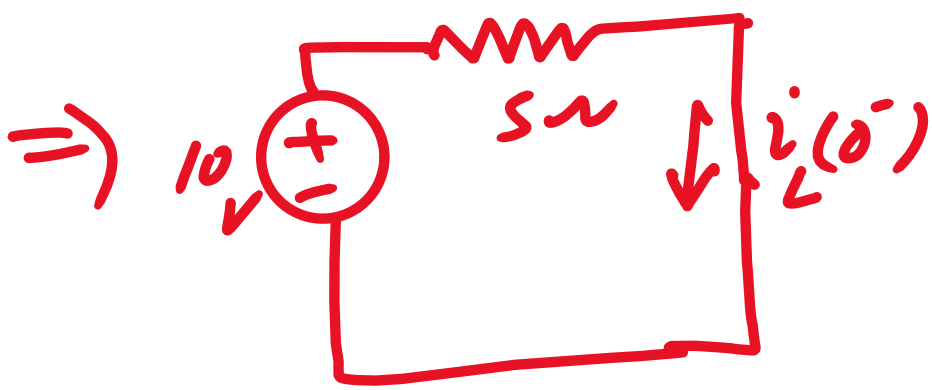
Before switching, the circuit has been connected for a long time, so it is in **steady state** at $t = 0^-$.

For DC steady state:

- An inductor behaves like a **short circuit**.
- The switch is **open** before $t = 0$.



1Ω will be shorted out



Hence the current through the inductor is

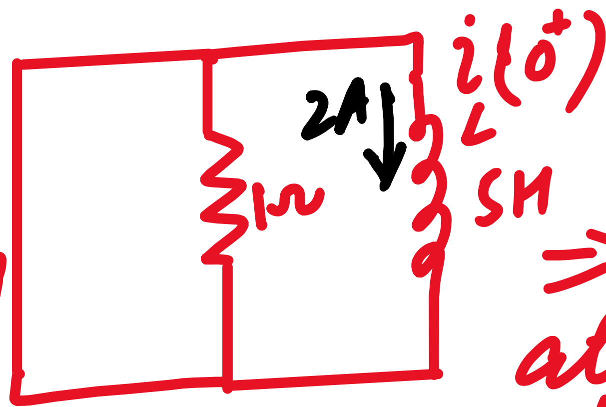
$$i_L(0^-) = \frac{10}{5} = 2A$$

Therefore,

$$\boxed{i_L(0^-) = 2A}$$

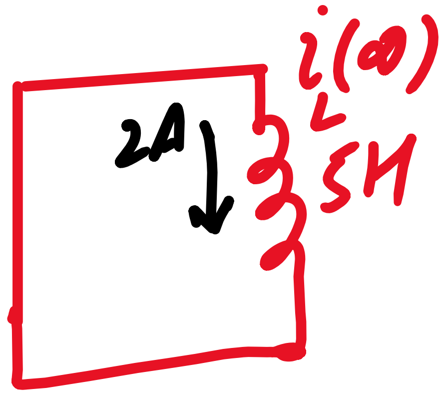
At $t \to \infty$: the circuit will look like

At $t=0$, switching happens,



, 1Ω will be shorted out

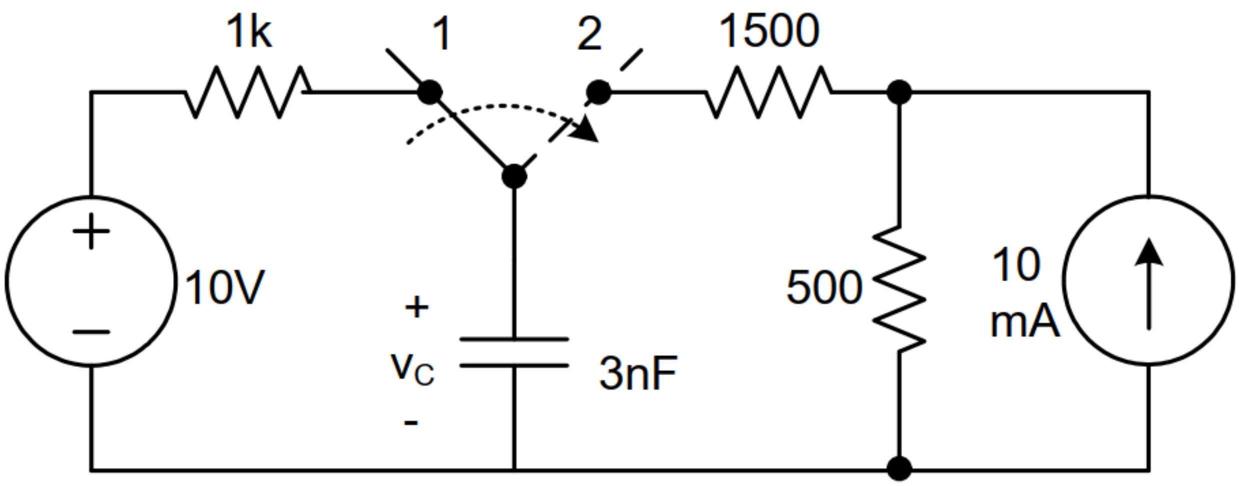
$\Rightarrow$ at $t=\infty$,



$$\boxed{i_L(\infty) = 2A}$$

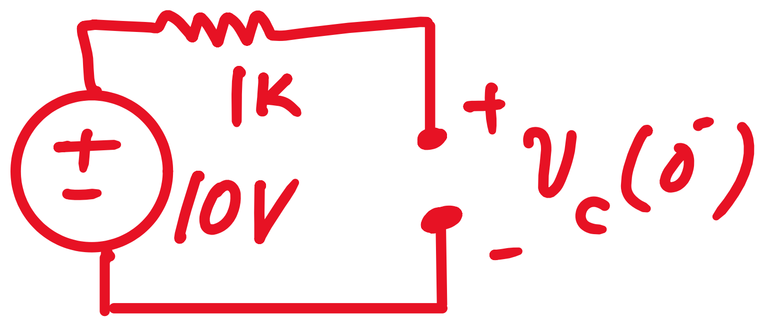
$i_L(t) = 2A$, for all time

as we don't have any dissipating element.



vC(0-) = ______V

vC(∞) = ______V



Before switching, the switch is at position **1** for a long time.

The capacitor is connected to the 10 V source through the $1\ k\Omega$resistor.

At steady state:

- capacitor behaves like an **open circuit**
- no current flows through the $1\ k\Omega$ resistor
- therefore, no voltage drops across the resistor

Hence the capacitor charges fully to the source voltage:

$$\boxed{v_C(0^-) = 10V}$$

After switching at t = 0s to position **2** and waiting a long time ($t \rightarrow \infty$):

Again, the capacitor behaves as an open circuit.

So, no current flows through the 1500Ω resistor either, meaning no voltage drop across it.

Therefore, the capacitor voltage equals the voltage at the right node.

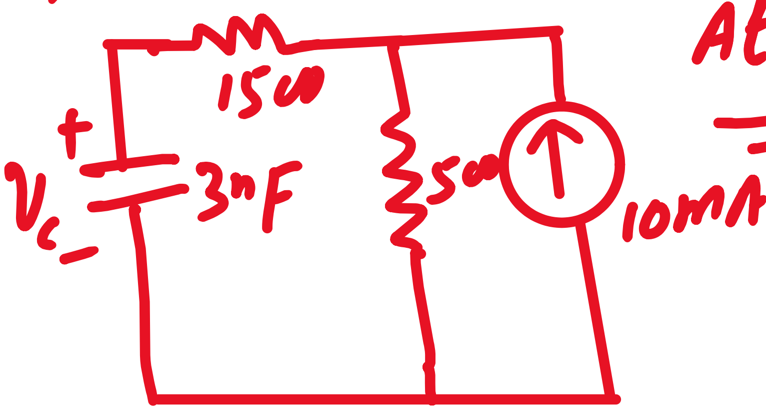
Now analyze the right-side network:

$$V = IR$$
$$V = (10\,mA)(500\Omega)$$
$$V = (0.01)(500) = 5V$$

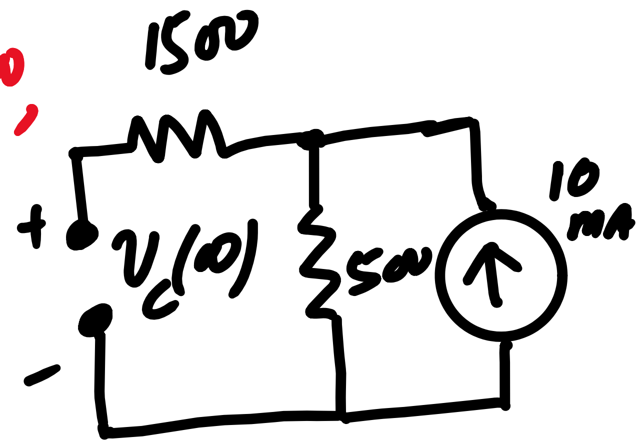
Thus:

$$\boxed{v_C(\infty) = 5V}$$

At $t = 0$,

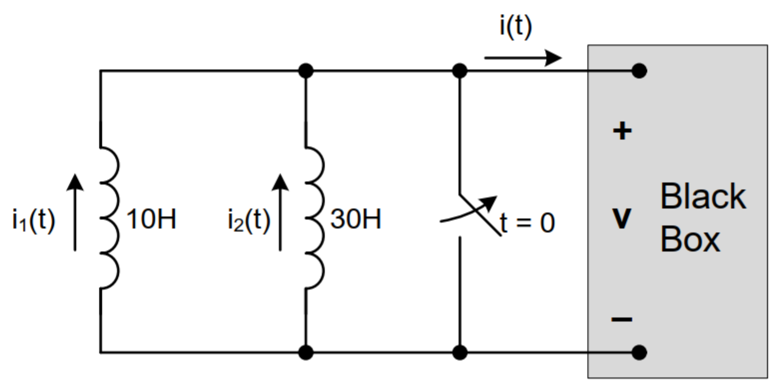


At $t = \infty$,



The two inductors in the figure below are connected across the terminals of a block box at **t = 0**. The resulting voltage **v** for **t > 0** is known to be **$-1800e^{-20t}$** V. It is also known that **$i_1(0) = 4A$** and **$i_2(0) = -16A$**.

a) Find **$i_1(t)$** for $t \geq 0$
b) Find **$i_2(t)$** for $t \geq 0$
c) How much **energy** is delivered to the black box in the time interval **$0 \leq t \leq \infty$?**
d) How much **energy** was initially stored in the parallel inductors?



Given:

$$v(t) = -1800e^{-20t} \text{ V}$$

For both inductors, current directions are **upward**, but voltage polarity is **+ at top, - at bottom.**

So current enters the **negative terminal** of each inductor. Therefore:

$$v(t) = -L\frac{di}{dt}$$

So,

$$\frac{di}{dt} = -\frac{v(t)}{L}$$

---

a) Find $i_1(t)$

For $L_1 = 10H$:

$$\frac{di_1}{dt} = -\frac{-1800e^{-20t}}{10}$$
$$\frac{di_1}{dt} = 180e^{-20t}$$

Integrate:

$$i_1(t) = i_1(0) + \int_0^t 180\, e^{-20x} dx$$
$$i_1(t) = 4 + 180\left[\frac{1 - e^{-20t}}{20}\right]$$
$$i_1(t) = 4 + 9(1 - e^{-20t})$$
$$\boxed{i_1(t) = 13 - 9e^{-20t}\ \text{A}}$$

---

b) Find $\boldsymbol{i_2(t)}$

For $L_2 = 30H$:

$$\frac{di_2}{dt} = -\frac{-1800e^{-20t}}{30}$$
$$\frac{di_2}{dt} = 60e^{-20t}$$
$$i_2(t) = i_2(0) + \int_0^t 60\, e^{-20x} dx$$
$$i_2(t) = -16 + 60\left[\frac{1 - e^{-20t}}{20}\right]$$
$$i_2(t) = -16 + 3(1 - e^{-20t})$$
$$\boxed{i_2(t) = -13 - 3e^{-20t}\ \text{A}}$$

---

c) Energy delivered to black box

Current, entering the black box:

$$i(t) = i_1 + i_2$$
$$i(t) = (13 - 9e^{-20t}) + (-13 - 3e^{-20t})$$
$$i(t) = -12e^{-20t}$$

Power delivered to black box:

$$p(t) = v(t)i(t)$$
$$p(t) = (-1800e^{-20t})(-12e^{-20t})$$
$$p(t) = 21600e^{-40t}$$

Energy:

$$W = \int_0^\infty 21600\, e^{-40t} dt$$
$$W = 21600 \left[\frac{1}{40}\right]$$
$$\boxed{W = 540\ \text{J}}$$

---

d) Initial energy stored in inductors

Formula:

$$W = \frac{1}{2} L i^2$$

For 10 H inductor:

$$W_1 = \frac{1}{2}(10)(4)^2 = 80J$$

For 30 H inductor:

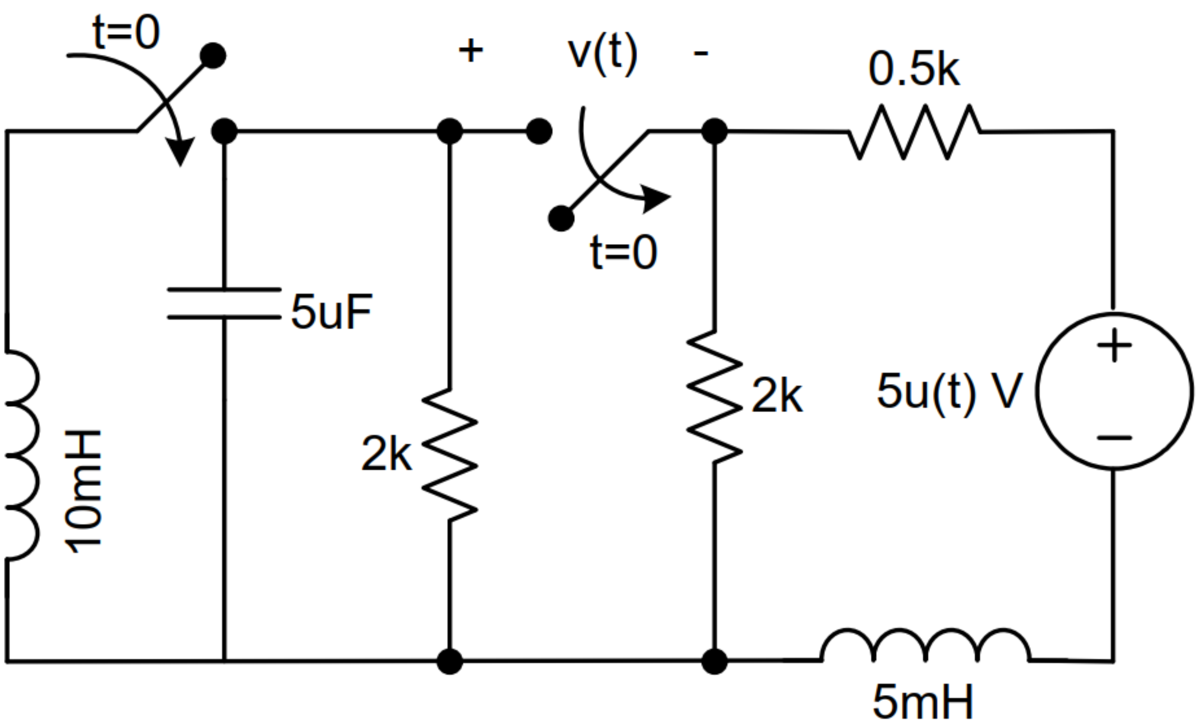
$$W_2 = \frac{1}{2}(30)(-16)^2$$
$$W_2 = 3840J$$

Total:

$$W_T = 80 + 3840$$
$$\boxed{W_T = 3920\ \text{J}}$$

Final answers:

$$\boxed{i_1(t) = 13 - 9e^{-20t}\ \text{A}}$$
$$\boxed{i_2(t) = -13 - 3e^{-20t}\ \text{A}}$$
$$\boxed{E_{\text{delivered}} = 540\ \text{J}}$$
$$\boxed{E_{\text{initial}} = 3920\ \text{J}}$$



Find the expression for v(t), > **0**, i.e., the **voltage across the switch** after it has **opened**. [Note: **5u(t)** means the voltage was **0** before **t = 0** and **5V** after it.] The two switches operate simultaneously as shown

For $t < 0$:

$$5u(t) = 0V$$

So, there is no source before switching. Therefore:

$$v_C(0^-) = 0$$
$$i_{L1}(0^-) = 0, i_{L2}(0^-) = 0$$

By continuity:

$$v_C(0^+) = 0$$
$$i_{L1}(0^+) = 0, i_{L2}(0^+) = 0$$

After $t = 0$, the switch opens, so:

$$v(t) = V_{\text{left}} - V_{\text{right}}$$

Left side

The left side contains $10mH$, $5\mu F$, and $2k\Omega$, but there is **no source**.

Also:

$$v_C(0^+) = 0, i_L(0^+) = 0$$

So, the left side has zero initial energy.

Therefore:

$$V_{\text{left}}(t) = 0$$

Right side

The right side is a first-order RL circuit.

At $t = 0^+$, the inductor acts as an open circuit:

$$V_{\text{right}}(0^+) = 0$$

At $t = \infty$, the inductor acts as a short circuit. So, using voltage divider:

$$V_{\text{right}}(\infty) = 5\left(\frac{2k}{0.5k + 2k}\right)$$
$$V_{\text{right}}(\infty) = 5\left(\frac{2000}{2500}\right)$$
$$V_{\text{right}}(\infty) = 4V$$

Now:

$$\tau = \frac{L}{R_{th}}$$
$$R_{th} = 0.5k + 2k = 2500\Omega$$
$$\tau = \frac{5mH}{2500}$$
$$\tau = \frac{0.005}{2500} = 2 \times 10^{-6} s$$

So:

$$V_{\text{right}}(t) = V(\infty) + [V(0^+) - V(\infty)]e^{-t/\tau}$$
$$V_{\text{right}}(t) = 4 + (0 - 4)e^{-t/(2\times10^{-6})}$$
$$V_{\text{right}}(t) = 4(1 - e^{-500000t})$$

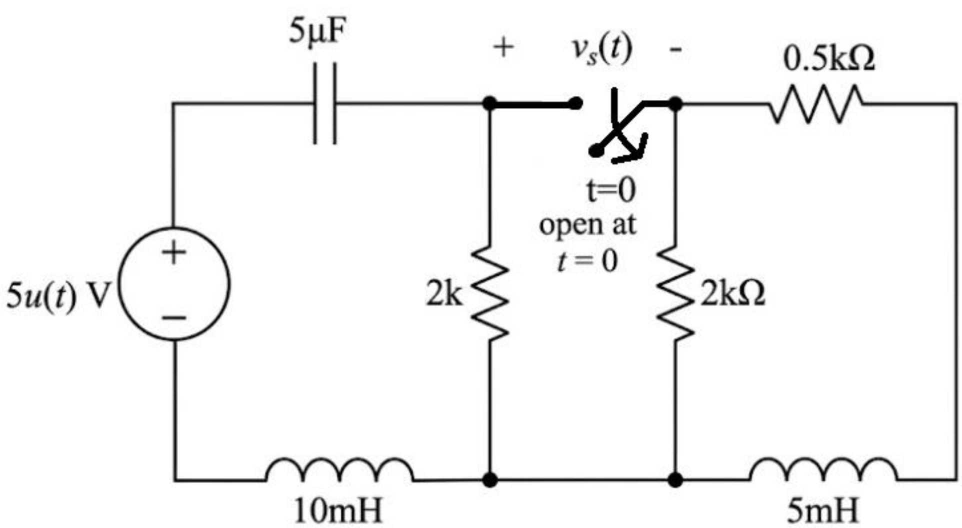
Since:

$$v(t) = V_{\text{left}} - V_{\text{right}}$$
$$v(t) = 0 - 4(1 - e^{-500000t})$$
$$\boxed{v(t) = -4(1 - e^{-500000t})V, t > 0}$$

or:

$$\boxed{v(t) = 4e^{-500000t} - 4\text{ V}}$$

Find the expression for $v_s(t)$, for $t > 0$, i.e., the **voltage across the switch** after it has **opened**. [Note: $5u(t)$ means the voltage was 0 before $t = 0$ and 5V after it.]



After switch opens:

$$v_s(t) = v_2(t) - v_1(t)$$

Right side has no source and no initial energy, so:

$$v_1(t) = 0$$

Therefore:

$$v_s(t) = v_2(t)$$

Now solve the left side as a **series RLC circuit**.

Given:

$$R = 2000\Omega$$
$$L = 10mH = 0.01H$$
$$C = 5\mu F = 5 \times 10^{-6}F$$

For series RLC:

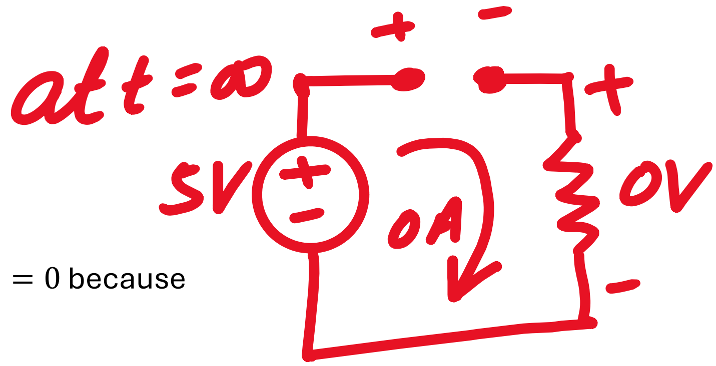
$$\alpha = \frac{R}{2L}$$
$$\alpha = \frac{2000}{2(0.01)}$$
$$\alpha = 100000\ s^{-1}$$

Also:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$\omega_0 = \frac{1}{\sqrt{(0.01)(5 \times 10^{-6})}}$$
$$\omega_0 = 4472\ s^{-1}$$

Since:

$$\alpha > \omega_0$$



the circuit is **overdamped**.

So:

$v_2(t) = v_2(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$, As $v_2(\infty) = 0$ because

Thus , $v_2(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

where:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
$$s_1 = -100000 + \sqrt{(100000)^2 - (4472)^2}$$
$$s_1 = -100.05$$
$$s_2 = -100000 - \sqrt{(100000)^2 - (4472)^2}$$
$$s_2 = -199899.95$$

So:

$$v_2(t) = A_1 e^{-100.05t} + A_2 e^{-199899.95t}$$

At $t = 0^+$:

$$v_2(0^+) = 0$$
$$A_1 + A_2 = 0$$

Now find derivative at $t = 0^+$.

At $t = 0^+$, capacitor behaves like a short circuit, and inductor current is initially zero. The full source voltage appears across the inductor:

$$v_L(0^+) = 5V$$
$$v_L = L\frac{di}{dt}$$
$$5 = 0.01\frac{di}{dt}$$
$$\frac{di}{dt} = 500\ A/s$$

Since:

$$v_2 = Ri$$
$$\frac{dv_2}{dt} = R\frac{di}{dt}$$
$$\frac{dv_2}{dt} = 2000(500)$$
$$\frac{dv_2}{dt} = 10^6\ V/s$$

Now differentiate:

$$\frac{dv_2}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

At $t = 0^+$:

$$A_1 s_1 + A_2 s_2 = 10^6$$

Since:

$$A_2 = -A_1$$
$$A_1(s_1 - s_2) = 10^6$$
$$A_1 = \frac{10^6}{-100.05 - (-199899.95)}$$
$$A_1 = 5.005$$

So:

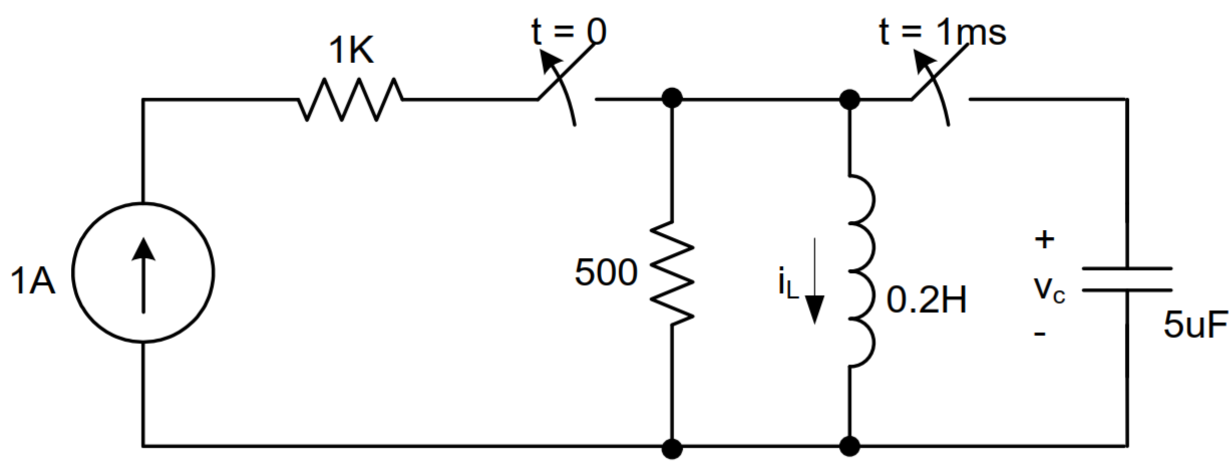
$$A_2 = -5.005$$

Therefore:

$$\boxed{v_s(t) = 5.005e^{-100.05t} - 5.005e^{-199899.95t}}$$
$$\boxed{v_s(t) = 5.005(e^{-100.05t} - e^{-199899.95t})V, t > 0}$$

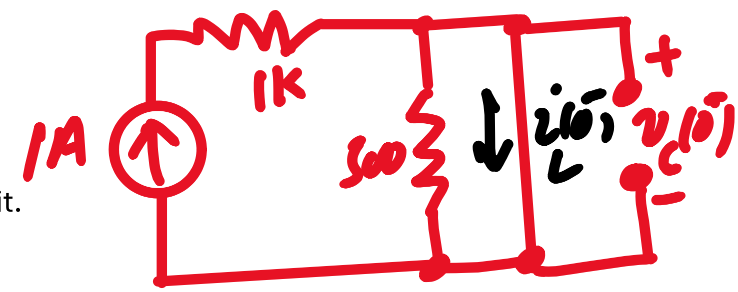
Approximately:

$$\boxed{v_s(t) \approx 5\left(e^{-100t} - e^{-2\times10^5 t}\right)V}$$



Find $i_L(t), t \geq 0^-$. Note the switches do not operate simultaneously.

1) Before $t = 0$



Left switch is closed; right switch is open.

At DC steady state, inductor acts like a short circuit.

So, all source current flows through the inductor:

$$\boxed{i_L(0^-) = 1A}$$

Also, $v_c(0^-) = 0V$
As capacitor is shorted out

Because inductor current cannot change suddenly:

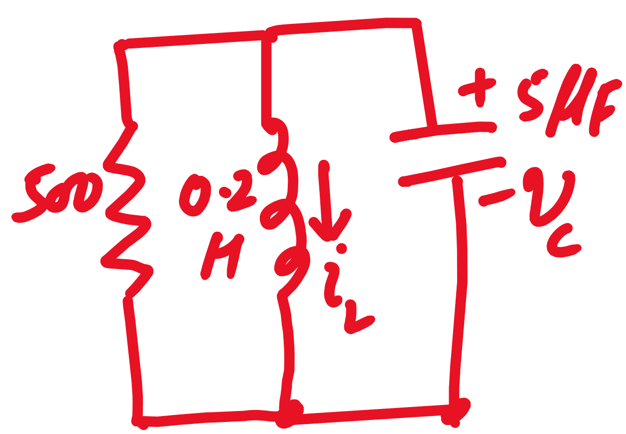
$$\boxed{i_L(0^+) = 1A}$$

$\Rightarrow v_c(0^-) = v_c(0^+) = 0V$

---

2) For $0 < t < 1ms$

At $t = 0$, left switch opens. The source is disconnected.



Now we have a parallel RLC circuit.

For parallel RLC:

$$\alpha = \frac{1}{2RC}$$

$$\alpha = \frac{1}{2(500)(5 \times 10^{-6})}$$
$$\alpha = 200\ rads^{-1}$$

Also:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$\omega_0 = \frac{1}{\sqrt{(0.2)(5 \times 10^{-6})}}$$
$$\omega_0 = 1000\ rads^{-1}$$

Since:

$$\alpha < \omega_0$$

the response is underdamped. As the response of UD CCT. Is given as

$\boldsymbol{i_L(t) = B_1 e^{-\alpha t} cos\omega_d t + B_2 e^{-\alpha t} sin\omega_d t}$ **-----------→ Eqn. 1**

Where,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$
$$\omega_d = \sqrt{1000^2 - 200^2}$$
$$\omega_d = 980\ rads^{-1}$$

Thus , Eqn. 1 becomes,

$\boldsymbol{i_L(t) = B_1 e^{-200t} cos980t + B_2 e^{-200t} sin980t}$ **------→ Eqn. 2**

Now, we need initial conditions to find $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$.

Condition 1: At $t = 0$, $i_L(0) = 1A$, the above equation becomes by putting these values,

$$\boldsymbol{B_1 = 0}$$

Condition 2: $\frac{di_L}{dt}\ at\ t = 0\ sec$

$$\frac{di_L(0^+)}{dt} = \frac{V_C(0^+)}{L} = \frac{0}{L} = 0$$

Differentiating Eqn. 2 , and putting above values, we get

$$0 = -200B_1 + 980B_2$$

$\Rightarrow B_2 = \frac{200}{980} = 0.204$, Putting the values of B_1 and B_2 in Eqn. 2, We get

$$\boxed{i_L(t) = e^{-200t}\cos\ 980t + 0.204\, e^{-200t}\sin\ 980t}$$

At $t = 1ms$: the above eqn. becomes

$$i_L(0.001) = e^{-0.2}\cos\,(0.98) + 0.204\, e^{-0.2}\sin\,(0.98)$$
$$= e^{-0.2}[\cos\,(0.98) + 0.204\sin\,(0.98)]$$

$$\boxed{i_L(1ms) = 0.6A}$$

---

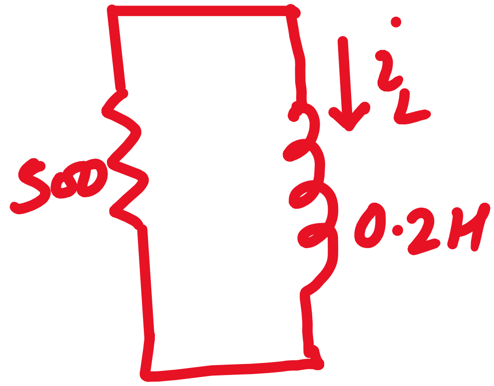
3) For $t > 1ms$

$\tau = \frac{L}{R} = \frac{0.2}{500} = 0.4ms,\ i_L(\infty) = 0A$ *as seen through the circuit*

$$i_L(t) = i_L(\infty) + [i_L(1ms) - i_L(\infty)]e^{-\frac{(t-t_0)}{\tau}}$$

$$i_L(t) = 0 + [0.6-0]e^{-\frac{(t-0.001)}{0.0004}} = 0.6\, e^{-2500t}e^{2.5}$$

$$\boxed{i_L(t) = 7.31e^{-2500t}, t > 1ms}$$



Final piecewise answer for $t \geq 0^- s$:

$$\boxed{i_L(t) = \begin{cases} 1A, & t = 0^- \\ e^{-200t}\cos\ 980t + 0.204\, e^{-200t}\sin 980t\ A & 0 \leq t < 1ms \\ 7.31e^{-2500t}A, & t \geq 1ms \end{cases}}$$

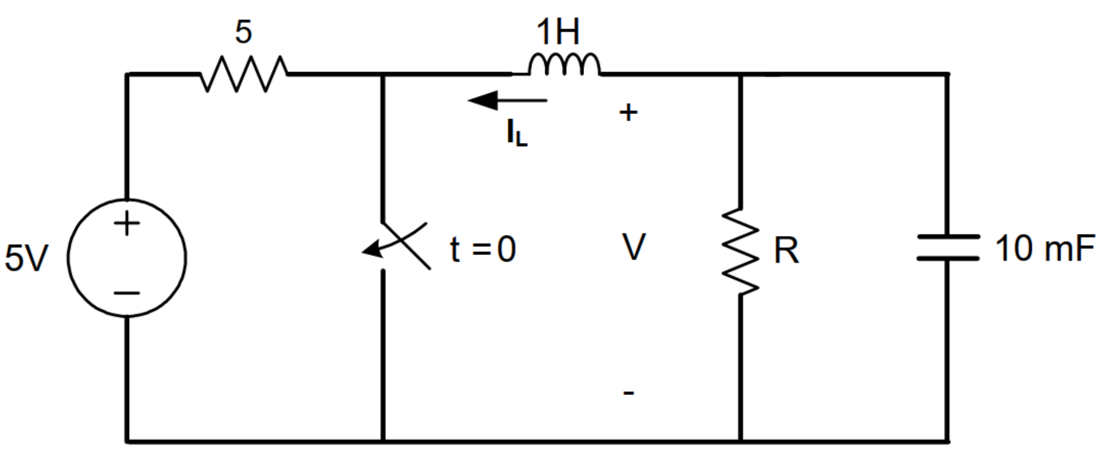
The circuit shown in Figure has been operating for a very long time. At t=0, Switch is closed. Find the voltage *v(t)* for t > 0 when R is equal to;

(a) 2 Ω

(b) 5 Ω

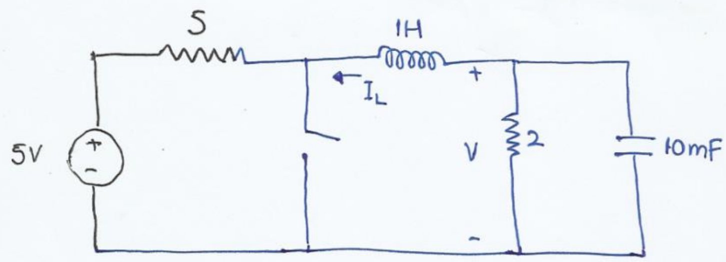
(c) 6 Ω

Characterize (name) the nature of response in each case.



(d) $R = 2\Omega$

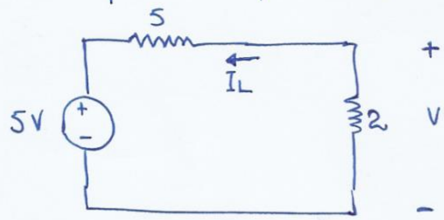
Initial Conditions can be calculated from the cct. $t < 0$



$\cancel{\alpha} \Rightarrow I_L(0-) = ?$

$V(0-) = ?$

Cap. will be Open cct & Ind. will be Short ccted.

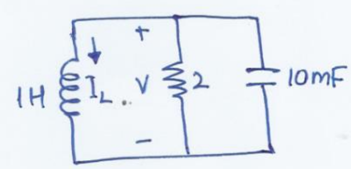


$I_L = \dfrac{0-5}{7} \Rightarrow \boxed{I_L(0-) = I_L(0+) = -714.3\text{ mA}}$

$V = -I_L(2) \Rightarrow \boxed{V(0-) = V(0+) = 1.43\text{ V}}$

At $t=0$, Switch: CLOSED. The cct. becomes Source-free Parallel RLC cct.

Computing α and ω_0



$$\alpha = \frac{1}{2RC} = \frac{1}{2(2)(10\times 10^{-3})} = 25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 10\times 10^{-3}}} = 10$$

Since $\alpha > \omega_0 \Rightarrow$ Overdamped response. So

$$\boxed{V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}}$$

where $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$= -25 \pm \sqrt{(25)^2 - (10)^2}$

$\boxed{S_1 = -2.09}$ $\boxed{S_2 = -48}$

Values of A_1 and A_2 need to be found from the derived initial conditions. So, at $t=0$

$$V(0) = A_1 + A_2 \quad \Rightarrow \quad A_1 + A_2 = 1.43 \text{ --- } (1)$$

To write other Eq.

$$i_c = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}\left[A_1 e^{s_1 t} + A_2 e^{s_2 t}\right]$$

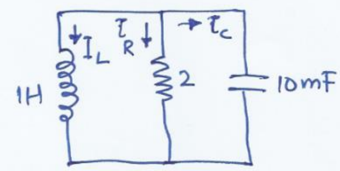
$$= C\left[A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}\right]$$

At $t=0$.

$$I_c(0) = C\left[A_1 s_1 + A_2 s_2\right]$$

$I_c(0)$? (after switch is CLOSED) KCL Eq.

$$-I_c(0+) - I_R(0+) - I_L(0+) = 0$$



or

$$I_c(0+) = -I_L(0+) - I_R(0+)$$

$$= -(-714.3\times10^{-3}) - \frac{V(0+)}{R}$$

$$= 714.3\times10^{-3} - \frac{1.43}{2}$$

$$\boxed{I_c(0+) = -0.7\text{ mA}}$$

So,

$$-0.7\times10^{-3} = C\left[A_1 s_1 + A_2 s_2\right]$$

Putting values of s_1 & s_2 & C.

$$\Rightarrow \quad -2.09A_1 - 48A_2 = -0.07$$

or

$$A_1 + 22.97A_2 = 0.0334 \text{ --- } (2)$$

Solving (1) and (2) simultaneously,

$$1.43 - A_2 = 0.0334 - 22.97A_2$$

$$\boxed{A_2 = -0.064}$$

$$A_1 = 1.43 - A_2$$
$$= 1.43 - (-0.064)$$

$$\boxed{A_1 = 1.5}$$

$$V(t) = 1.5\, e^{-2.09t} - 0.064\, e^{-48t} \Rightarrow \text{Ans.}$$

(b) $R = 5\Omega$

$\alpha = 10$

$\omega_o = 10$ (Same as before)

Since $\alpha = \omega_o \Rightarrow$ Critically damped response. So,

$$\boxed{V(t) = e^{-\alpha t}\left(A_1 t + A_2\right)}$$

where A_1 and A_2 can be found based on the initial Conditions. Please note that initial Conditions have to be found again because Value of R is changed now. However, all the procedure to find the initial Conditions and A_1 and A_2 is exactly Same as Part (a).

(c) $R = 6\Omega$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(6)(10\times10^{-3})} = 8.33$$

$\omega_o = 10$ (same as before)

Since $\alpha < \omega_o \Rightarrow$ Underdamped response. So,

$$\boxed{V(t) = e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right)}$$

where $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

where B_1 and B_2 can be easily found based on the initial Conditions (to be determined again)

For above Question (a), estimate the Settling time. You do not need to use an iterative approach to find the exact value. Simply consider the term which accounts for settling the system response and neglect the term which decays earlier.

From Q3 (a), as calculated

$$V(t) = 1.5\, e^{-2.09t} - 0.064\, e^{-48t}$$

Settling time:

- Time required for the response to remain 1% of its max. value
- So, we need to find the max. absolute value of $V(t)$. For that, differentiating $V(t)$ w.r.t. time and putting it to Zero.

$$\frac{d}{dt} v(t) = 0$$

$$(1.5)(-2.09)\, e^{-2.09t} - (0.064)(-48)\, e^{-48t} = 0$$

$$+3.135\, e^{-2.09t} = 3.072\, e^{-48t}$$

or $$0.98\, e^{-45.91t} = 1 \qquad (\ln e = 1)$$

$$\Rightarrow \boxed{t \approx 0 \text{ Sec}}$$

(Time for max. $V(t)$)
NOT Settling Time

Max. amplitude of $V(t)$ can be found as

$$V(0) = 1.5 - 0.064$$

$$\boxed{V(0) = 1.436\ V}$$

$$1\% \text{ of max. value} = \frac{1}{100}(1.436) = 0.01436\ V$$

Time to attain this max. value = Settling time = ?

Again using

$$V(t) = 1.5\, e^{-2.09t} - 0.064\, e^{-48t}$$

or $$0.01436 = 1.5\, e^{-2.09t} - 0.064\, e^{-48t}$$

To find 't', this equation can be solved using an iterative approach. More simply, consider only the term involving '$e^{-2.09t}$', since this term decays later on (compared to e^{-48t}) and thus the settling time is mainly determined by '$e^{-2.09t}$'.

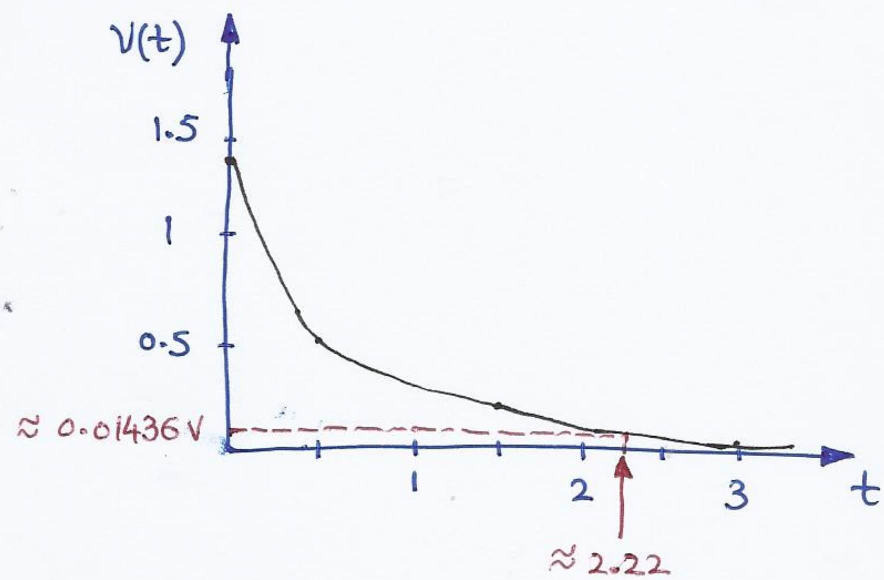
$$0.01436 \approx 1.5e^{-2.09t}$$

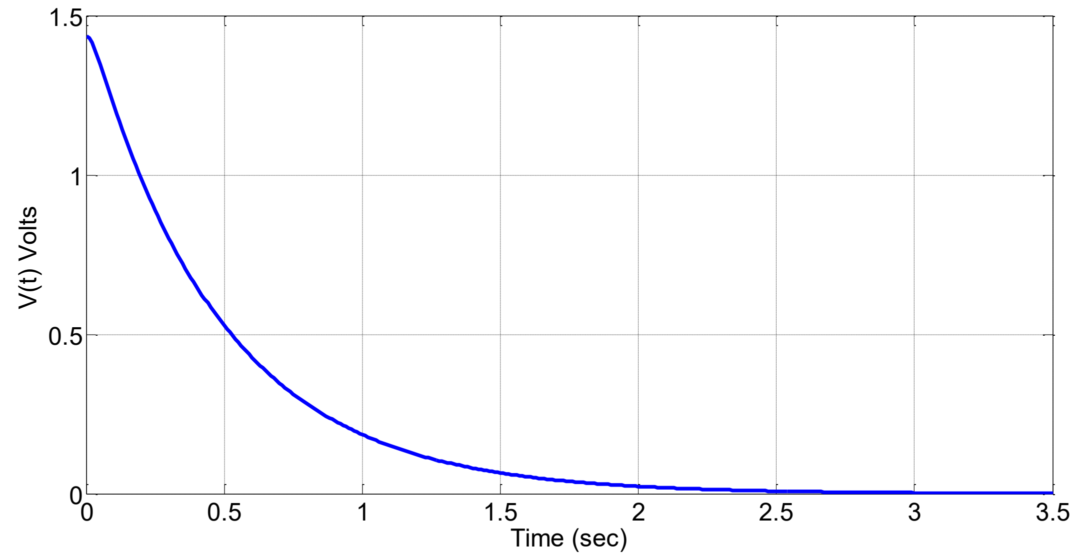
or $$e^{-2.09t} \approx 0.00957$$

$$-2.09t \ln e \approx \ln(0.00957) \qquad (\ln e = 1)$$

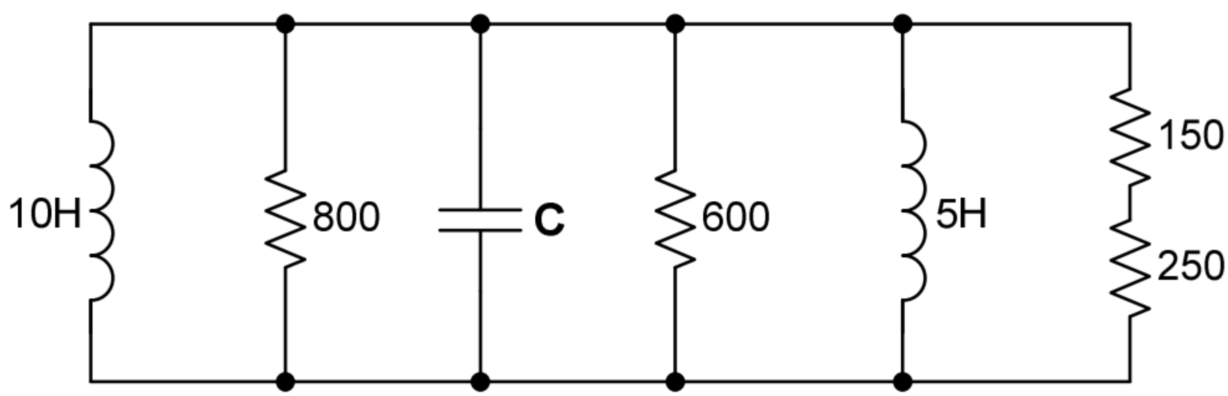
$$-2.09t \approx -4.648$$

$$\boxed{t \approx 2.22 \text{ Sec}} \quad \text{(Settling Time)}$$





For the circuit shown below, what should be the range of capacitance values 'C' for which the circuit will be underdamped?



For underdamped case,

$$\alpha^2 < w_o{}^2$$

$$\alpha = \frac{1}{2R_{eq}C}$$

$$w_o = \frac{1}{\sqrt{L_{eq}C}}$$

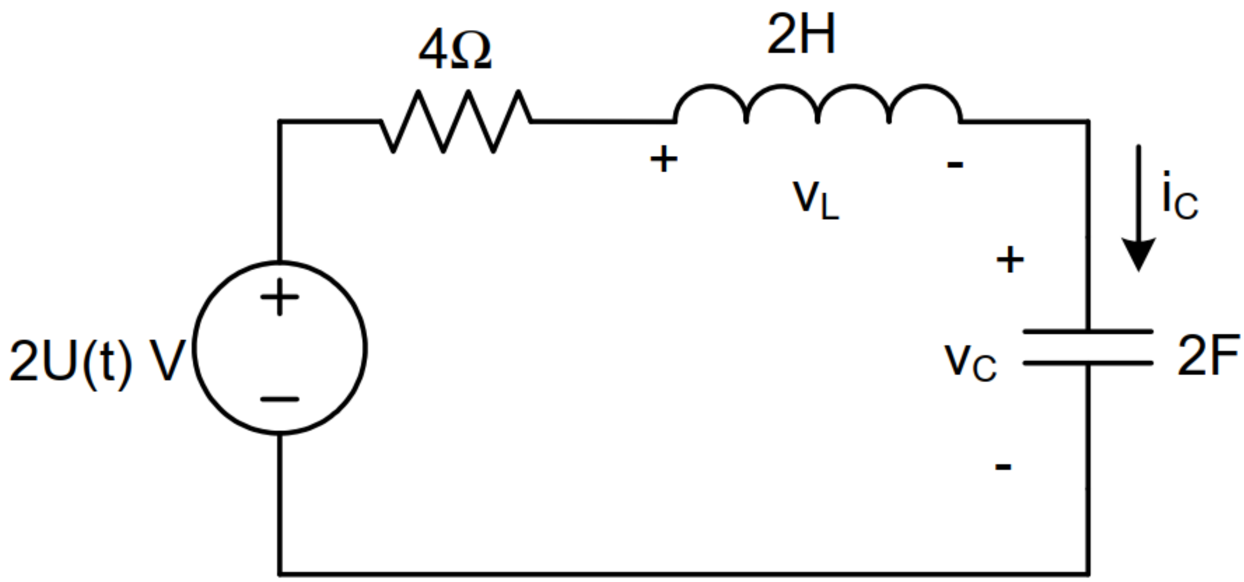
$$R_{eq} = 800||600||(150 + 250) = 184.615\Omega$$

$$L_{eq} = 10||5 = 3.333H$$

$$C > \frac{L_{eq}}{4R_{eq}{}^2}$$

$$C > 24.45\mu F$$

For the following network, in how much time does the capacitor charges to 1V? The system is assumed to be in steady state (operating for a long time) for $t < 0 \; and \; v_L(0^-) = 2V$, and $I_C(0^-) = 3A$ while all other initial conditions are zero (how these initial values were obtained is not of concern).

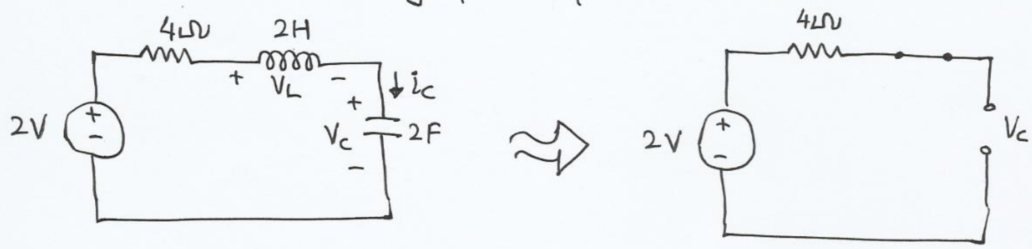


Q4:

After Switching, the cct. is a Driven RLC Series Circuit. So,

$$\boxed{V_c(t) = V_c(\infty) + \text{Sol. of natural response}}$$

Where $V_c(\infty)$ can be easily found from the cct



$$\Rightarrow \boxed{V_c(\infty) = 2\,V}$$

To find the natural response, we need to Calculate

$$\alpha = \frac{R}{2L} = \frac{4}{2(2)} = 1 \text{ sec}^{-1}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.2}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \text{ rad/sec}$$

Since $\alpha > \omega_o \Rightarrow$ Over damped Case

Thus

$$\boxed{V_c(t) = 2 + A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

$$\left.\begin{matrix} A_1 \\ A_2 \\ S_1 \\ S_2 \end{matrix}\right\} ?$$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$= -1 \pm \sqrt{1 - \frac{1}{4}}$$

$$-1 \pm \frac{\sqrt{3}}{2} \qquad \boxed{S_1 = -0.134 \times -1.86}$$

Thus

$$\boxed{V_c(t) = 2 + A_1 e^{-0.134t} + A_2 e^{-1.86t}}$$

where A_1 and A_2 need to be found based on initial Conditio

At $t=0$

$V_c(0) = 2 + A_1 + A_2$ $\qquad$ $V_c(0+) = V_c(0-) = 0\,V$ (Give in Q. !)

$\Rightarrow \boxed{A_1 + A_2 = -2}$ —— (EQ. 1)

We need another Equation involving A_1 and A_2.

$$i_c = C \frac{d}{dt} V_c$$

$$= 2 \frac{d}{dt}\left[2 + A_1 e^{-0.134t} + A_2 e^{-1.86t}\right]$$

$$= 2\left[-0.134 A_1 e^{-0.134t} - 1.86 A_2 e^{-1.86t}\right]$$

At $t=0$

$$i_c(0) = 2\left[-0.134 A_1 - 1.86 A_2\right]$$

Now $i_c(0) = i_L(0) \Rightarrow$

$i_L(0+) = i_L(0-) = 0\,A$ For inductor (Give in Q !)

So,

$\boxed{+0.134 A_1 + 1.86 A_2 = 0}$ —— (EQ. 2)

Solving (1) and (2) Simultaneously

From (1)

$$0.134 A_1 + 0.134 A_2 = -0.268 \quad \text{—— (1)}$$

$$\overset{+}{\underset{-}{}}\ 0.134 A_1 \overset{+}{\underset{-}{}} 1.86 A_2 = 0 \quad \text{—— (2)}$$

$$-1.726\ A_2 = -0.268 \Rightarrow$$

$$\boxed{A_2 = 0.155}$$

$$A_1 = -2 - A_2$$

$$= -2 - 0.155$$

$$\boxed{A_1 = -2.155}$$

Finally,

$$\boxed{V_c(t) = 2 + 0.155 e^{-0.134t} - 2.155 e^{-1.86t}}$$

As per requirements given in the Question we need to find how much time is taken by the Capacitor to charge to 1V.

i.e.

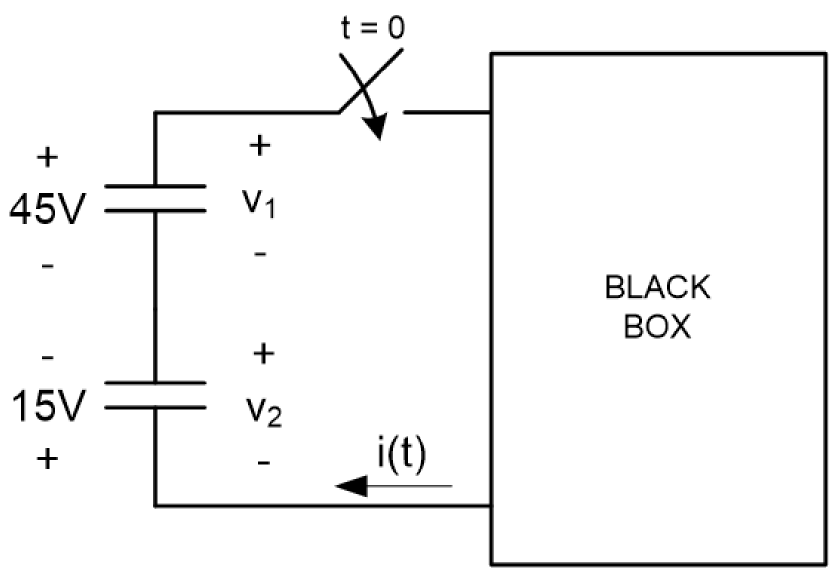
$$1 = 2 + 0.155\,e^{-0.134t} - 2.155\,e^{-1.86t}$$

$$\Rightarrow \quad 0.155\,e^{-0.134t} - 2.155\,e^{-1.86t} = -1$$

One Eq., one unknown :) So, it should be possible (in principle) to find 't'.

$$\boxed{t = 0.338 \text{ Sec}}$$

The current i(t) for t > 0 is known to be $\mathbf{900e^{-2500t}}$ **μA**. Write correct formulas for $\mathbf{v_1(t)}$ and $\mathbf{v_2(t)}$. Do not solve them.



Current enters the **negative terminal** of both capacitors, so use:

$$i(t) = -C\frac{dv}{dt}$$

$$v(t) = -\frac{1}{\mathrm{C}}\int_0^t i(t)\ d\tau + v_1(0^+)$$

Given:

$$i(t) = 900e^{-2500t}\ \mu A$$

For capacitor C_1:

$$\boxed{v_1(t) = 45 - \frac{1}{C_1}\int_0^t 900\ e^{-2500\tau}\ \mu A\ d\tau}$$

For capacitor C_2, note carefully:

The given 15 V polarity is opposite to v_2, therefore:

$$v_2(0) = -15V$$

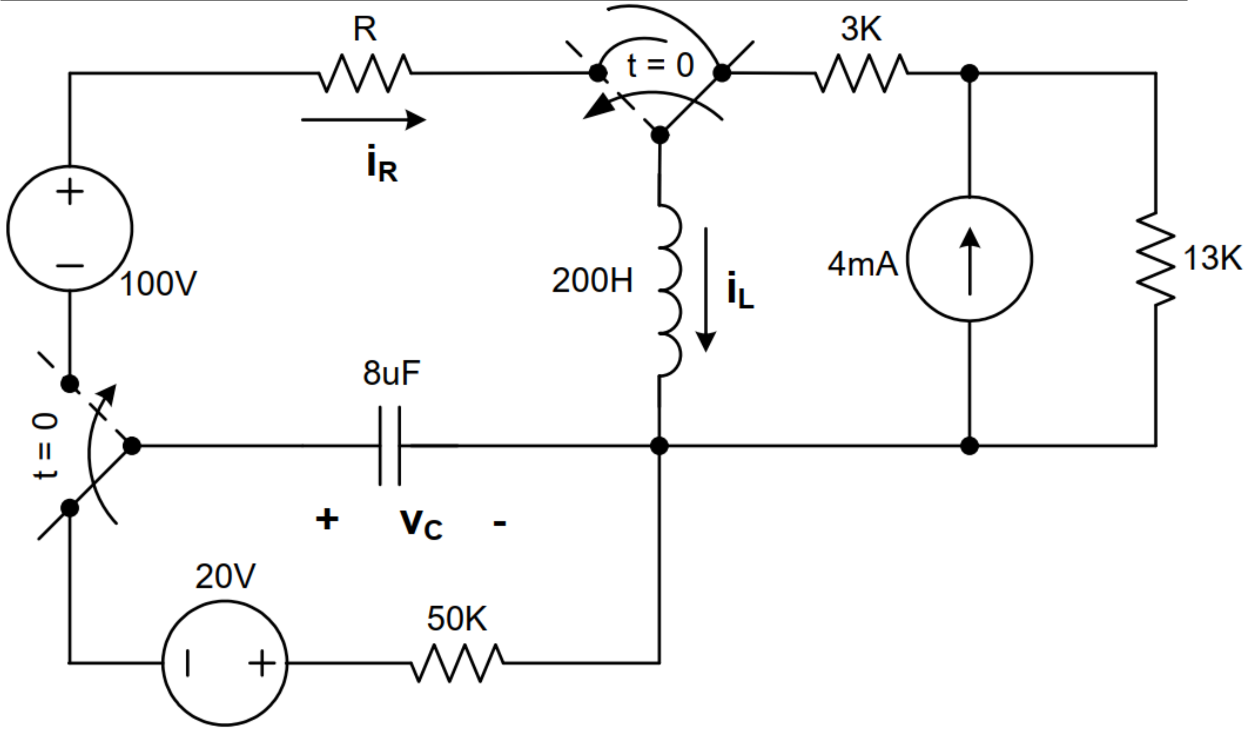
So:

$$\boxed{v_2(t) = -\frac{1}{C_2}\int_0^t 900\ e^{-2500\tau}\ \mu A\ d\tau - 15}$$

Here $C_1$and C_2should be in μF.

Given that the circuit below is to be used in **critically damped** configuration:

i)	Find the precise value of **R**?
ii)	What is **i_R** **before** and **after** the switching time at **t = 0**?



For critically damped case, $\alpha^2 = w_o{}^2$

$$\alpha = \frac{R}{2L}$$

$$w_o = \frac{1}{\sqrt{LC}}$$

i) $R = \frac{2L}{\sqrt{LC}} = 10k\Omega$

$$\alpha = 25rad/s$$

Current through Rbefore switching

Before $t = 0$, resistor $R$is disconnected from the inductor side, so no closed path exists through R.

$$\boxed{i_R(0^-) = 0A}$$

Current through Rjust after switching

Inductor current cannot change instantly:

$$i_L(0^+) = i_L(0^-)$$

Before switching, the inductor acts as a short circuit at DC. So, the $4mA$current source divides between $3k\Omega$ and $13k\Omega$.

Current through $3k\Omega$, which becomes inductor current:

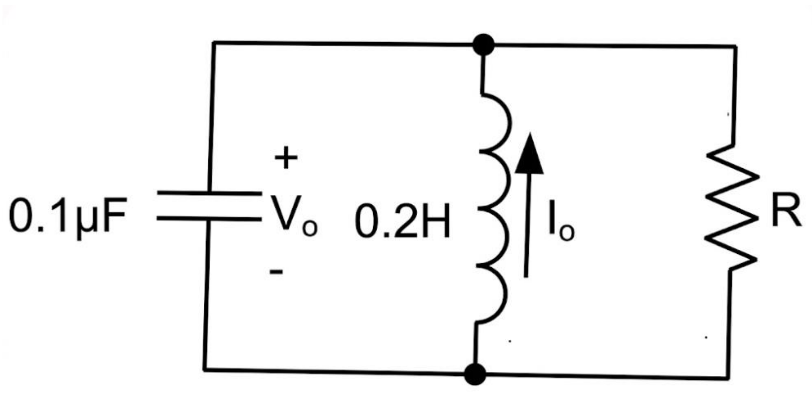
$$i_L(0^-) = 4mA\left(\frac{13k}{3k + 13k}\right)$$
$$i_L(0^-) = 4mA\left(\frac{13}{16}\right)$$
$$i_L(0^-) = 3.25mA$$

After switching, $R$is in series with the inductor, so:

$$\boxed{i_R(0^+) = i_L(0^+) = 3.25mA}$$

Final answers

$$\boxed{R = 10k\Omega}$$
$$\boxed{i_R(0^-) = 0A}$$
$$\boxed{i_R(0^+) = 3.25mA}$$



For the natural response circuit shown below, the voltage across the capacitor V_o at $t = 0^-$ is $10\ V$ and the current I_o through the inductor at $t = 0^-$ is $2\ A$. Determine the value (or range of values) of resistance R for which the voltage $v(t)$ across the capacitor (with the same polarity as V_o) will be **underdamped**.

For a **parallel RLC natural response**:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

Compare with:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

So,

$$2\alpha = \frac{1}{RC}$$
$$\alpha = \frac{1}{2RC}$$

and

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

For **underdamped response**:

$$\alpha < \omega_0$$

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}}$$
$$R > \frac{1}{2}\sqrt{\frac{L}{C}}$$

Given:

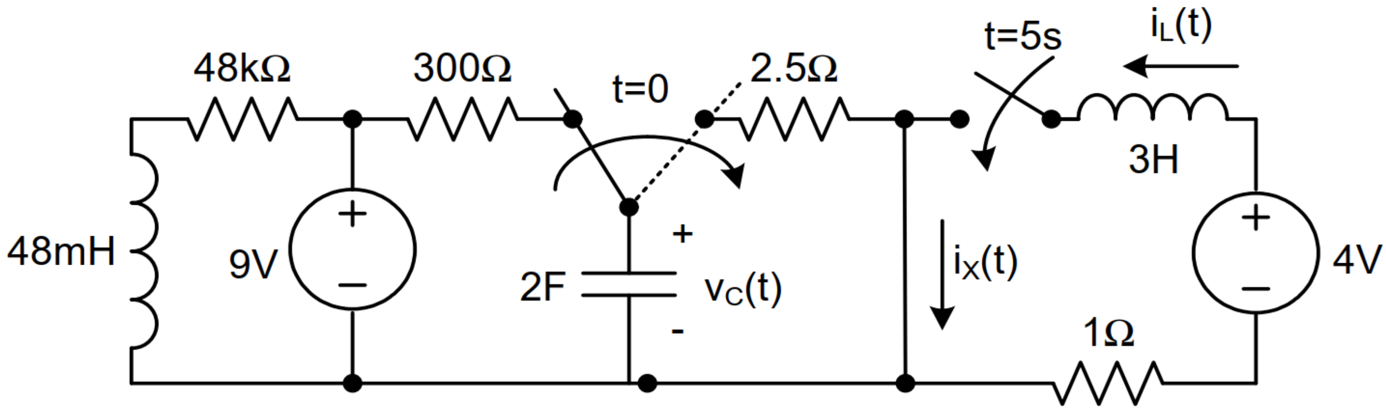
$$L = 0.2H$$
$$C = 0.1\mu F = 0.1 \times 10^{-6} = 10^{-7}F$$
$$R > \frac{1}{2}\sqrt{\frac{0.2}{10^{-7}}}$$
$$R > \frac{1}{2}\sqrt{2 \times 10^{6}}$$
$$R > \frac{1}{2}(1414.2)$$
$$R > 707.1\Omega$$

Therefore, the capacitor voltage $v(t)$will be **underdamped** for:

$$\boxed{R > 707\Omega}$$

At $R = 707\Omega$, it is critically damped.
For $R < 707\Omega$, it is overdamped.

For the circuit and switches configuration as below (all behaviour clearly shown), **find** expression for $i_X(t), for\ t > 0$. Note that the circuit is assumed to be operating for a long time before $t = 0$.



$$v_C(0^-) = 9V = v_C(0^+)$$

When the switching at t=0 occurs, a source-free RC circuit is created, so:

$$v_C(t) = v_C(0^+)e^{-\frac{t}{\tau}} = 9e^{-\frac{t}{5}}\,V, \qquad t > 0, \qquad as\ \tau = RC = 2.5 \times 2 = 5s$$

The current through the 2.5Ω resistor (towards right), that makes up current $i_X(t), 0 < t < 5$, will be

$$i_X(t) = \frac{v_C(t)}{2.5} = \frac{9e^{-\frac{t}{5}}}{2.5} = 3.6e^{-\frac{t}{5}}\,A, \qquad 0 < t < 5s$$

At $t = 5s$ another switching introduces a forced RL circuit to make up part of current $i_X(t)$. For the forced RL circuit, that begins operation at $t = 5s$.

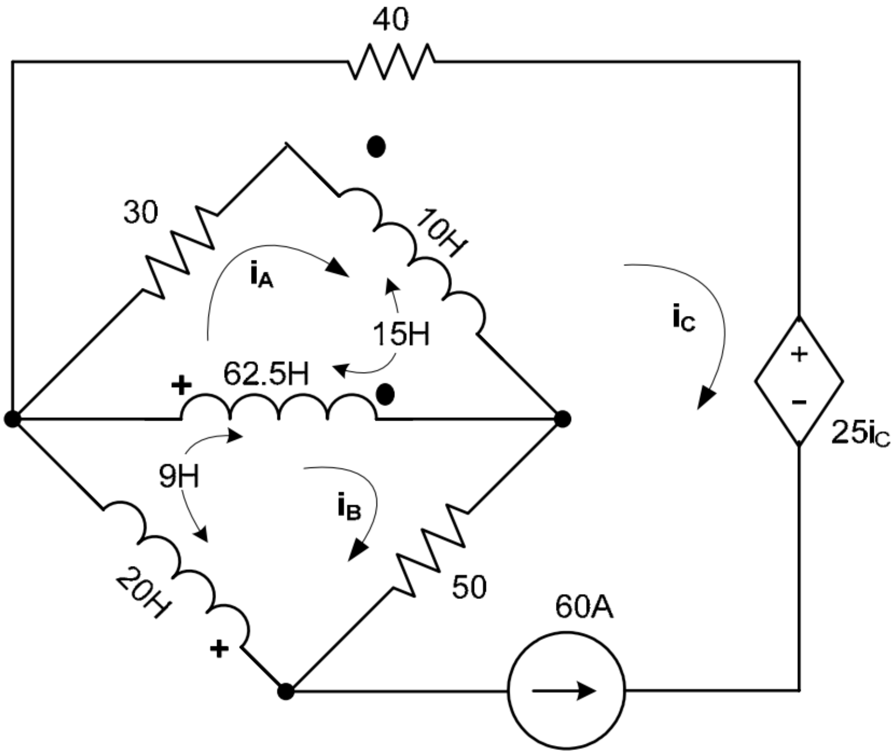
$$i_L(5^-) = i_X(5^+) = 0A, \qquad but\ i_L(\infty) = \frac{4}{1} = 4A$$

$$i_L(t) = i_L(\infty) + [i_L(5^+) - i_L(\infty)]e^{-\frac{t-5}{\tau}}u(t-5) = 4 + [0-4]e^{-\frac{t-5}{3}}u(t-5), as\ \tau = \frac{L}{R} = \frac{3}{1}s$$

Thus

$$i_X(t) = \begin{cases} 0, & t < 0 \\ 3.6e^{-\frac{t}{5}}\,A, & 0 < t < 5s \\ 3.6e^{-\frac{t}{5}} + \left[4 - 46e^{-\frac{t-5}{3}}\right]A, & t > 5s \end{cases}$$

Write all the Mesh Current equations and constraint equations using MCA technique. All equations must be in terms of the given names/labels of the currents marked



Loop a:

$$0 = 30(i_A - i_C) + 10\frac{d}{dt}(i_A - i_C) + 62.5\frac{d}{dt}(i_A - i_B) + 15\frac{d}{dt}(i_A - i_B) + 15\frac{d}{dt}(i_A - i_C) - 9\frac{d}{dt}i_B$$

Loop b:

$$0 = 50(i_B - i_C) + 20\frac{d}{dt}i_B + 62.5\frac{d}{dt}(i_B - i_A) + 9\frac{d}{dt}(i_B - i_A) + 9\frac{d}{dt}(i_B) + 15\frac{d}{dt}(i_C - i_A)$$

Loop c:

$$i_C = -60A$$

TABLE 9.1 Summary of Relevant Equations for Source-Free *RLC* Circuits

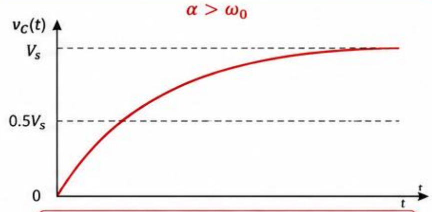
Condition	Criteria	α	ω_0	Response
Overdamped	$\alpha > \omega_0$	$\frac{1}{2RC}$ (parallel) $\frac{R}{2L}$ (series)	$\frac{1}{\sqrt{LC}}$	$A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
Critically damped	$\alpha = \omega_0$	$\frac{1}{2RC}$ (parallel) $\frac{R}{2L}$ (series)	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1 t + A_2)$
Underdamped	$\alpha < \omega_0$	$\frac{1}{2RC}$ (parallel) $\frac{R}{2L}$ (series)	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

COMPARISON OF RLC CIRCUIT RESPONSES

STEP RESPONSE (Source Applied at $t = 0$)

1) OVERDAMPED RESPONSE

$\alpha > \omega_0$



$v_C(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

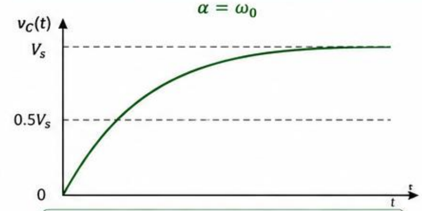
where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

(two distinct real roots)

- No oscillation
- Slowest response
- Takes longer to reach steady state
- Two real negative poles

2) CRITICALLY DAMPED RESPONSE

$\alpha = \omega_0$

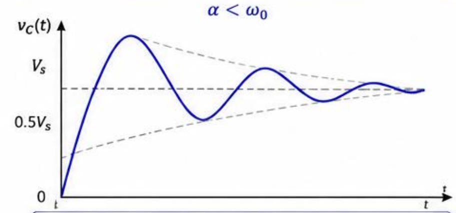


$v_C(t) = V_s + (A_1 + A_2 t)e^{-\alpha t}$

- No oscillation
- Fastest response without overshoot
- Ideal damping
- Repeated real pole ($s = -\alpha$)

3) UNDERDAMPED RESPONSE

$\alpha < \omega_0$



$v_C(t) = V_s + e^{-\alpha t}\ (A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

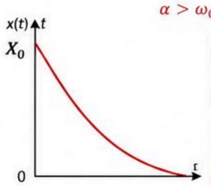
(complex conjugate roots)

- Oscillatory response
- Exponentially decaying sinusoid
- Moderate speed
- Complex conjugate poles

DISCHARGE RESPONSE (Source Removed, Initial Energy in L or C)

1) OVERDAMPED RESPONSE

$\alpha > \omega_0$



$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

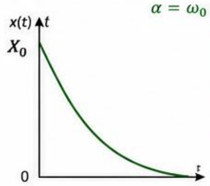
where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

(two distinct real roots)

- No oscillation
- Slow decay
- Two real negative poles

2) CRITICALLY DAMPED RESPONSE

$\alpha = \omega_0$

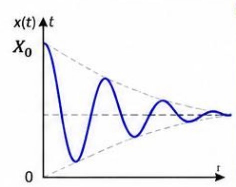


$x(t) = (A_1 + A_2 t)e^{-\alpha t}$

- No oscillation
- Fastest decay to zero
- Repeated real pole ($s = -\alpha$)

3) UNDERDAMPED RESPONSE

$\alpha < \omega_0$



$x(t) = e^{-\alpha t}\ (A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

(complex conjugate roots)

- Oscillatory decay
- Exponentially decaying sinusoid
- Complex conjugate poles

KEY PARAMETERS

$\alpha = \frac{R}{2L}$ (Damping factor)

$\omega_0 = \frac{1}{\sqrt{LC}}$ (Natural undamped frequency)

R, L, C are resistance, inductance and capacitance.

SUMMARY COMPARISON

Type	Condition	Poles	Oscillation	Step Response	Discharge Response
Overdamped	$\alpha > \omega_0$	Two real distinct	No	Slow rise, no overshoot	Slow decay, no oscillation
Critically Damped	$\alpha = \omega_0$	Real repeated	No	Fastest rise, no overshoot	Fastest decay, no oscillation
Underdamped	$\alpha < \omega_0$	Complex conjugate	Yes	Oscillatory with overshoot	Oscillatory decay to zero